BACCALAURÉAT - Session 2014

Epreuve de Discipline Non Linguistique

Mathématiques/Anglais

Benford's Law

<u>Leading digit</u>

The leading digit of 42 583.69 is 4; the leading digit of 0. 000 321 is 3.
If you look at the leading digit in a list of random numbers and then try to predict the frequency of each leading digit, you expect the result to be pretty close to 11% each. (i.e. one digit out of nine)

<u>Population</u>

According to INSEE (the French National Institute for Statistics and Economic Studies), the population in each town and village in "départments": 07, 26, 38, 73 and 74 and their leading digits are as follows:

Leading digit	Number of towns	Relative frequency	Benford's law
1	540		0.301
2	317		0.176
3	203		0.125
4	181		0.097
5	160		0.079
6	134		0.087
7	106		0.058
8	117		0.051
9	82		0.046
TOTAL	1840		

<u>Benford's law</u>

Benford's law, also called the **first-digit law**, refers to the frequency distribution of digits in many <u>real-life sources of data</u>. In the case of naturally-occurring data sets, digit 1 occurs as the first digit about 30% of the time. Digit 2 occurs less frequently than digit 1 and so on with larger digits occurring less often. This result has been found to apply to a wide variety of data sets, including lengths of rivers, street addresses, population numbers, electricity bills… It tends to be most accurate when values are distributed across multiple orders of magnitude.

This law is named after **Frank Benford**, a physicist who stated it in 1938. Besides, Benford's law is used in the accounting profession to detect fraud. Auditors can use it as a high-level check of a data set. If there are anomalies, it may be worth investigating more closely as a case of potential fraud.

Adapted from Wikipedia (http://en.wikipedia.org/wiki/Benford%27s_law)

Questions

1) Explain why digit 1 might be expected to occur as the leading digit about 11% of the time.
2) Complete the relative frequency column of the table about population.
   Did you find the expected 11%?
3) Compare your results with those of Benford's law.
4) Show how this law can be used as a practical tool in other areas.
5) Can you give other examples when mathematics have useful applications or other situations when nature seems to follow a mathematical rule?
6) Benford's law states: the probability of leading digit d appearing is $P(d) = \dfrac{\ln\left(\dfrac{d+1}{d}\right)}{\ln(10)}$

   $d \in \{1, 2, \ldots 9\}$. Prove that the sum of P(1) + P(2) + … + P(9) = 1 (using the formula of $P(d)$)