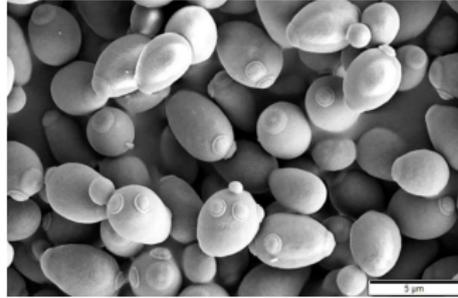
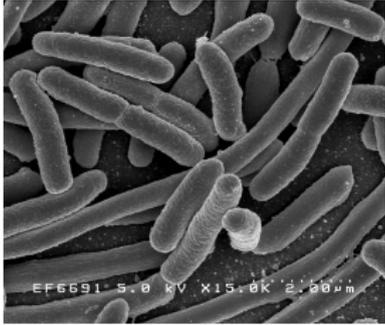


Inria

Modélisation mathématique des populations de cellules

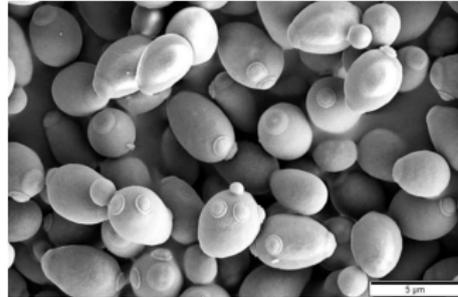
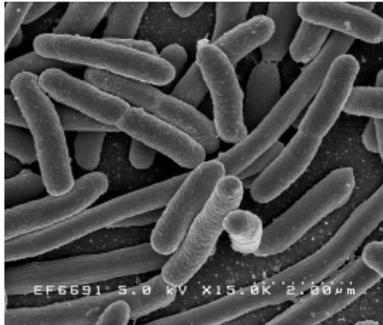
A. Marguet

◆ Différents organismes



◆ Bactéries : *Escherichia Coli* ◆ Levures : *Saccharomyces cerevisiae*

◆ Différents organismes



◆ Bactéries : *Escherichia Coli* ◆ Levures : *Saccharomyces cerevisiae*

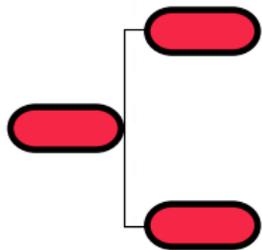
◆ Différentes applications

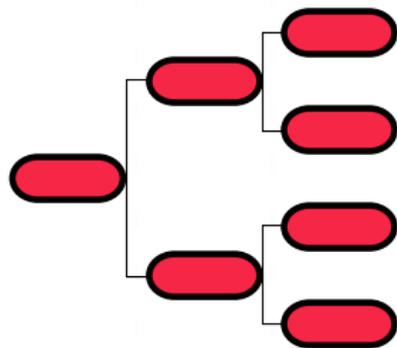


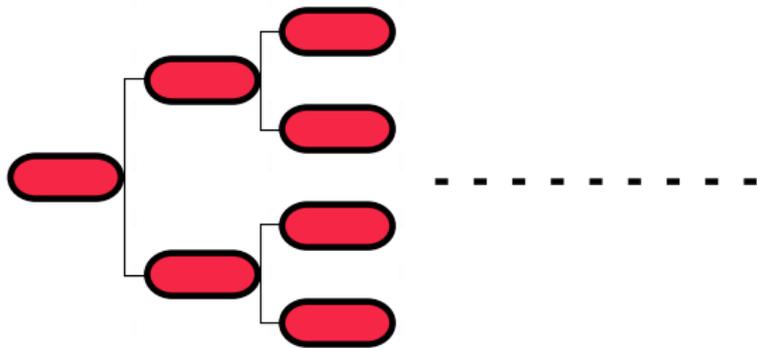
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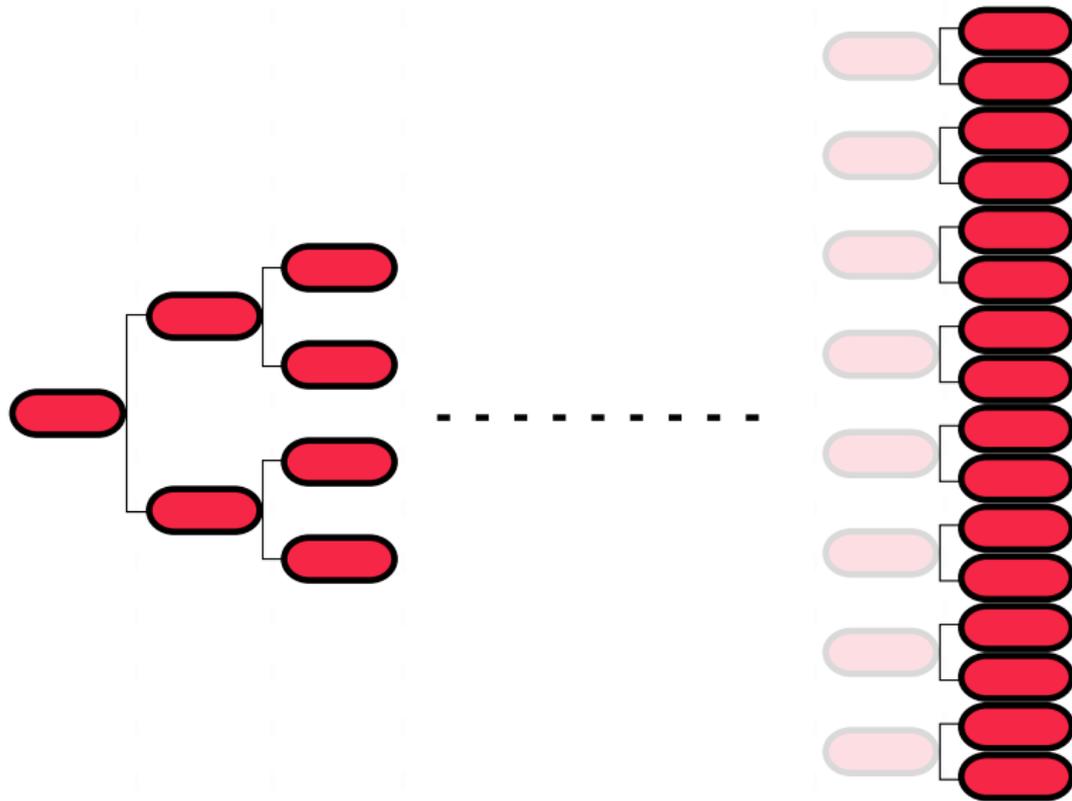
La croissance











On note X_n le nombre de cellules à la génération n .

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$$X_1 =$$

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$$X_1 = 1$$

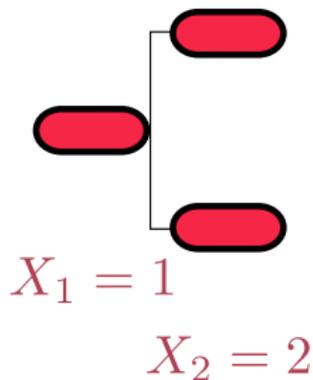
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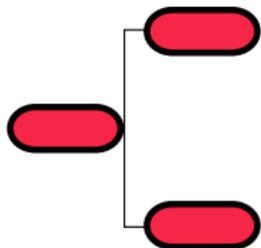
$$X_1 = 1$$

$$X_2 =$$

On note X_n le nombre de cellules à la génération n .



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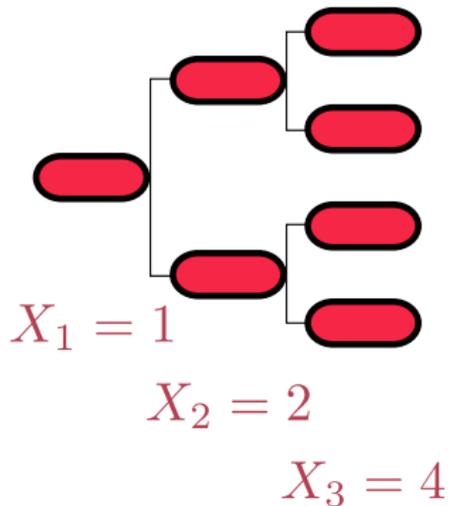


$$X_1 = 1$$

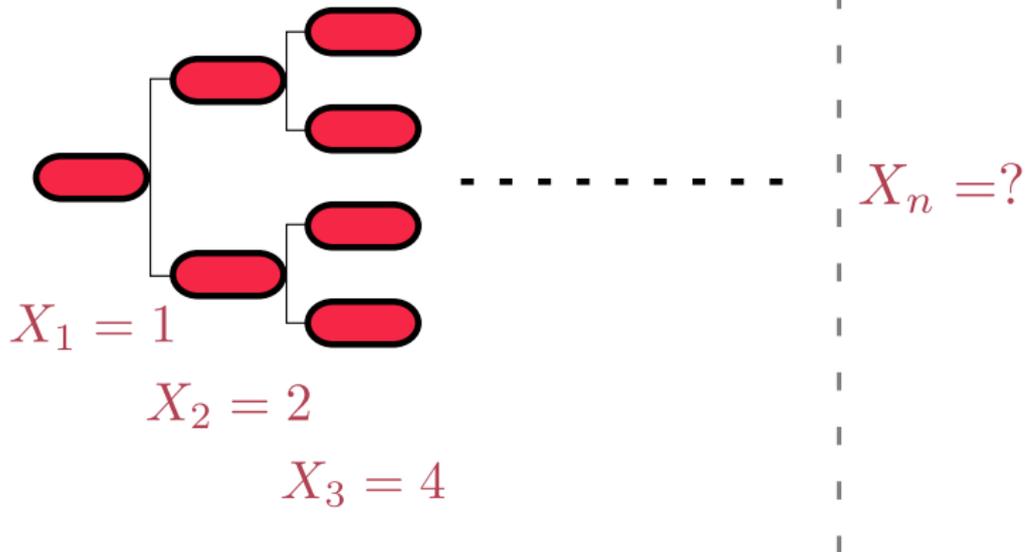
$$X_2 = 2$$

$$X_3 =$$

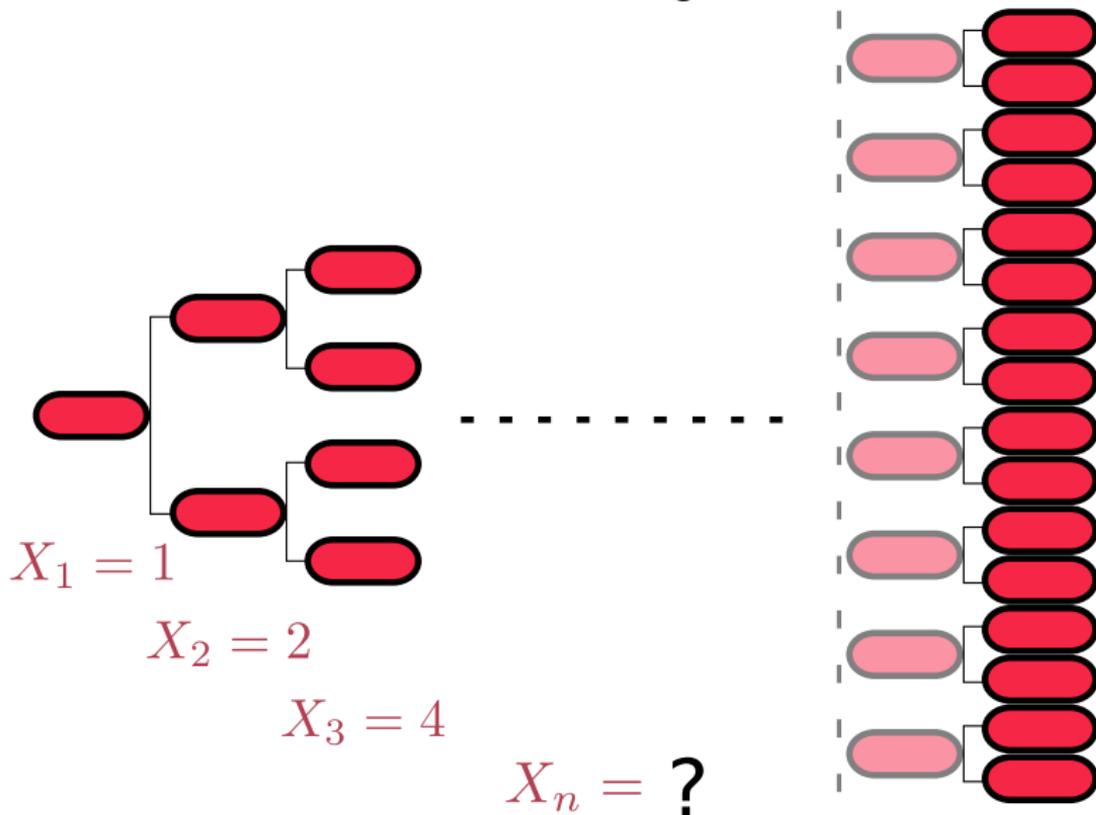
On note X_n le nombre de cellules à la génération n .



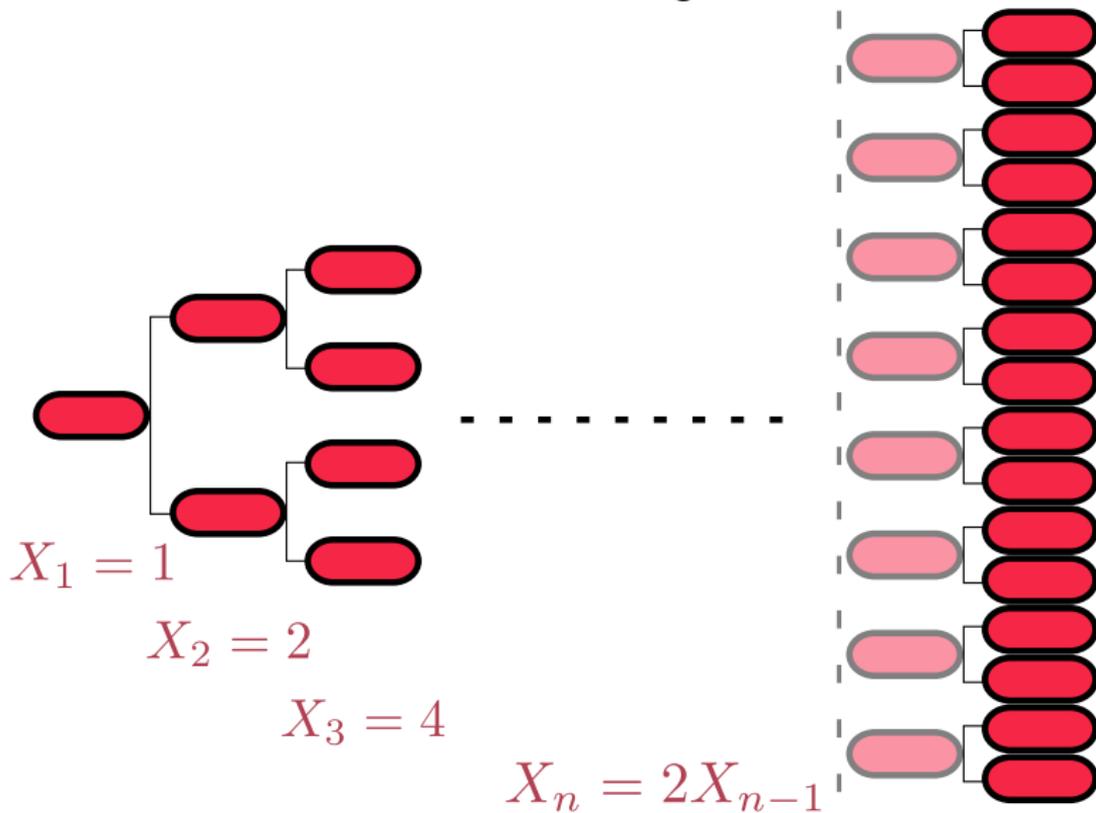
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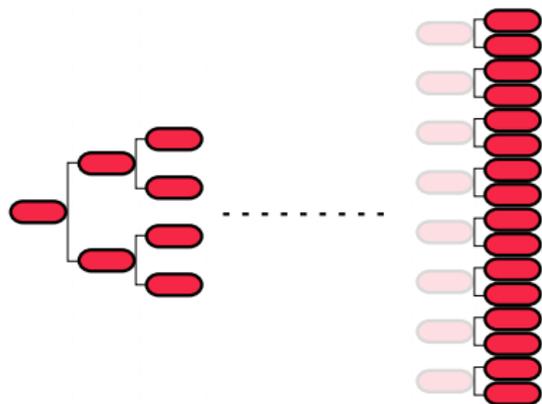


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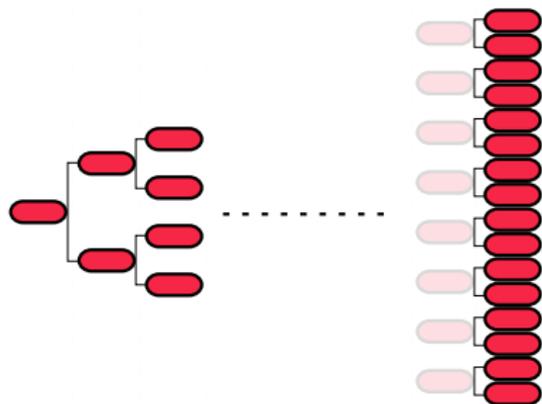


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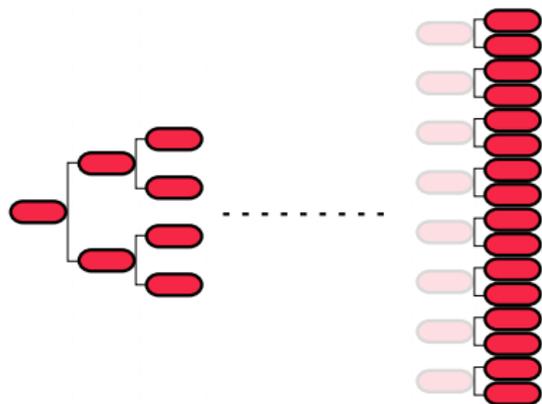




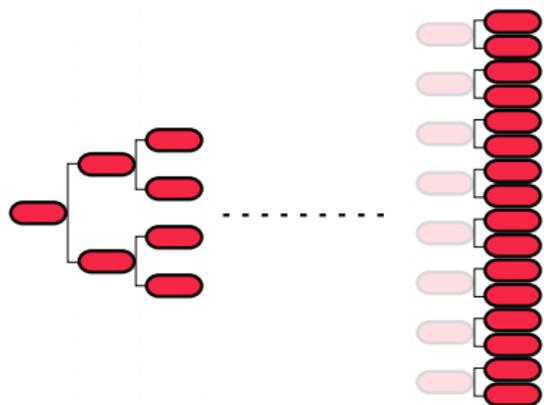
$$X_n = 2 \times X_{n-1}$$



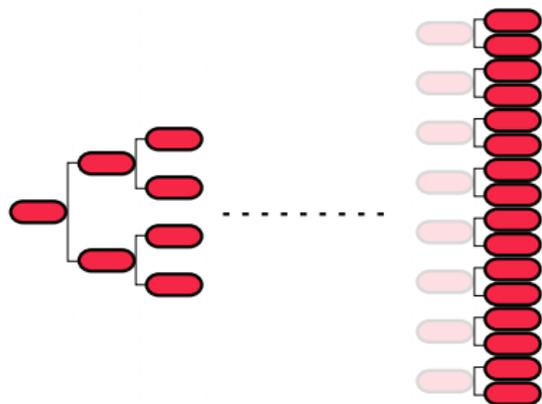
$$X_n = 2 \times X_{n-1} = 2 \times 2 \times X_{n-2}$$



$$X_n = 2 \times X_{n-1} = 2 \times 2 \times X_{n-2} = 2 \times 2 \times 2 \times X_{n-3}$$

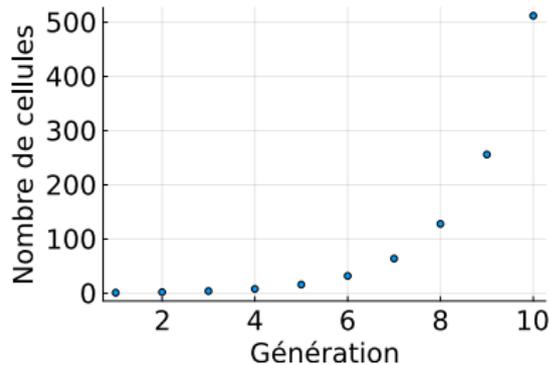
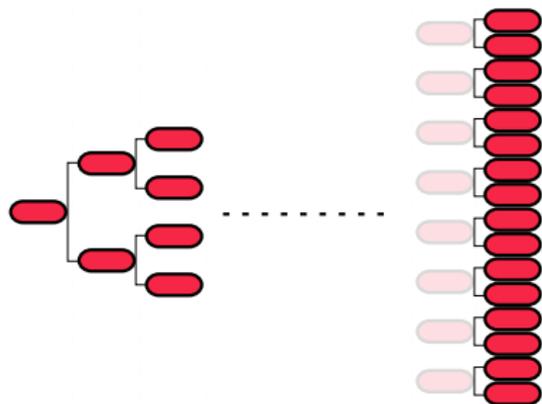


$$\begin{aligned}
 X_n &= 2 \times X_{n-1} = 2 \times 2 \times X_{n-2} = 2 \times 2 \times 2 \times X_{n-3} \\
 &= \underbrace{2 \times 2 \times \dots \times 2}_{n-1 \text{ fois}} \times X_1
 \end{aligned}$$



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$$X_n = 2^{n-1}$$

02

Échantillonnage

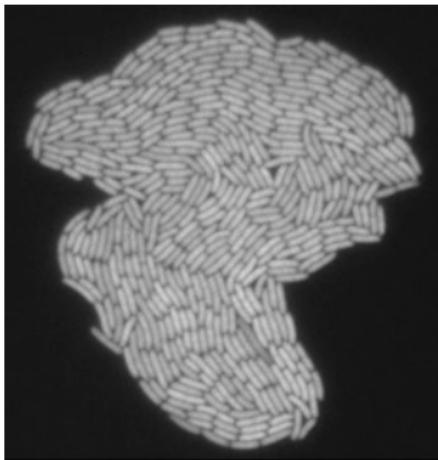
OPEN ACCESS PEER-REVIEWED

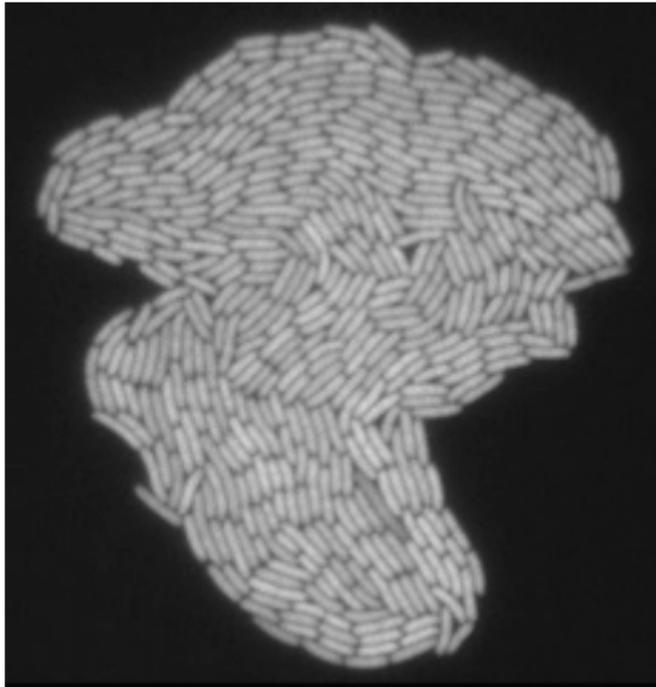
RESEARCH ARTICLE

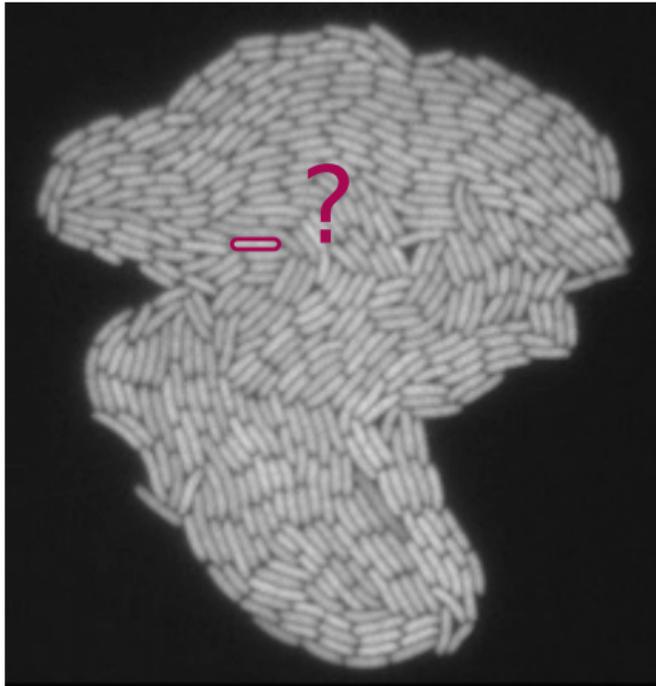
Aging and Death in an Organism That Reproduces by Morphologically Symmetric Division

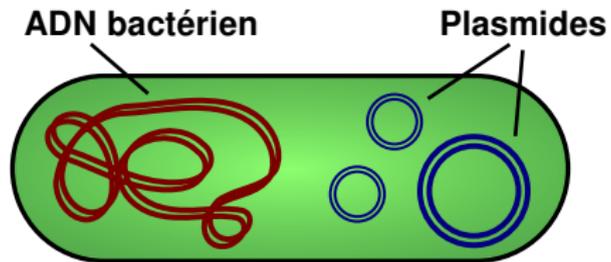
Eric J Stewart  Richard Madden, Gregory Paul, François Taddei

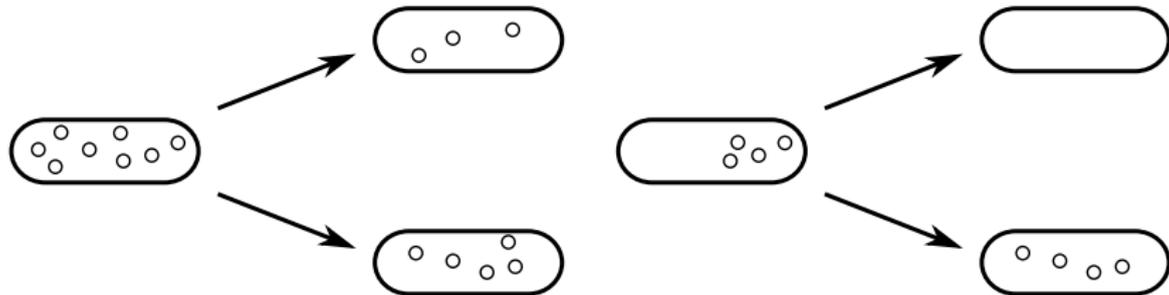
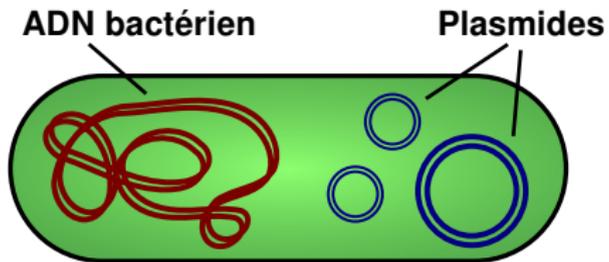
Published: February 1, 2005 • <https://doi.org/10.1371/journal.pbio.0030045>



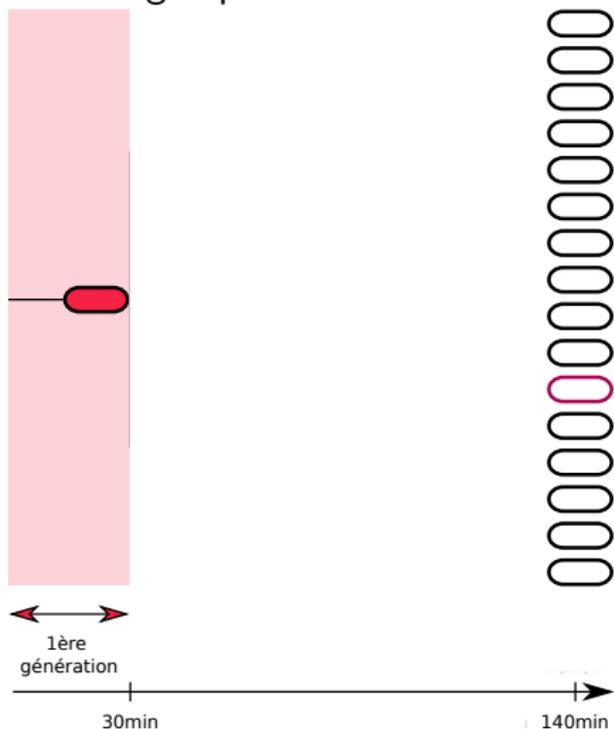
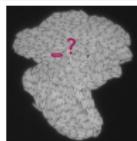




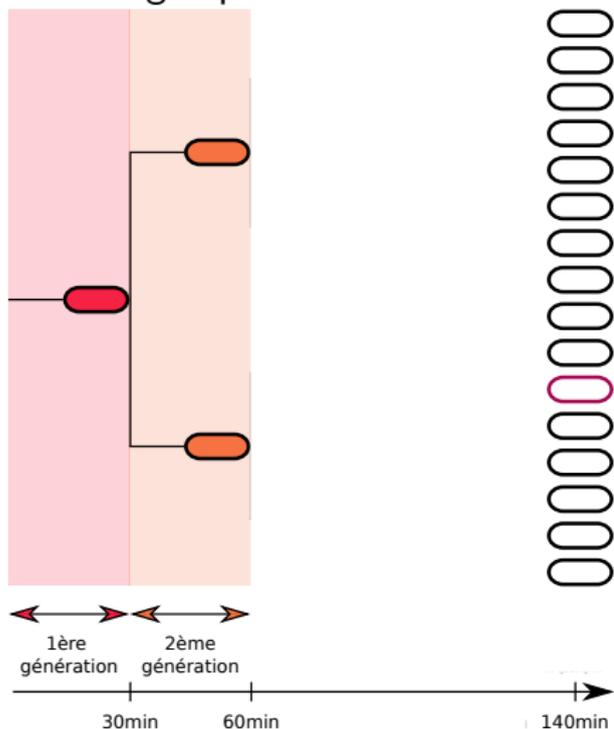
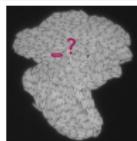




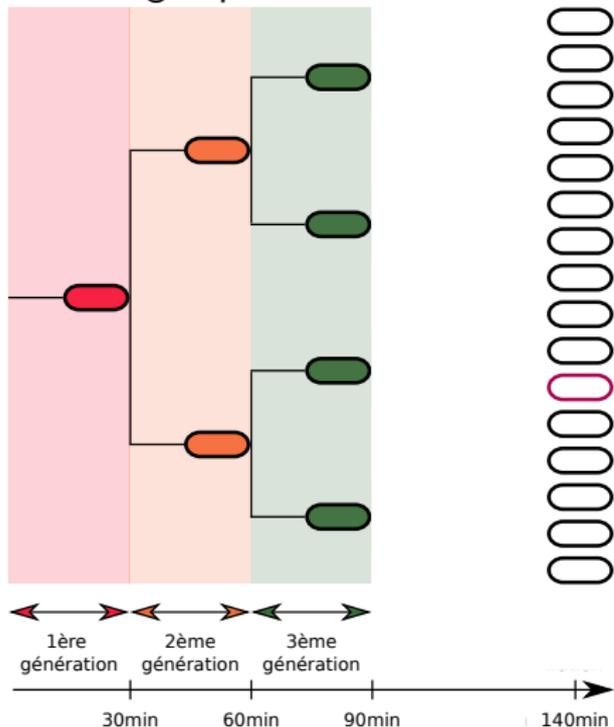
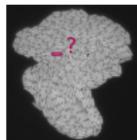
- ◆ Durée moyenne d'un cycle cellulaire : 30min.
- ◆ Échantillonnage après 140min.



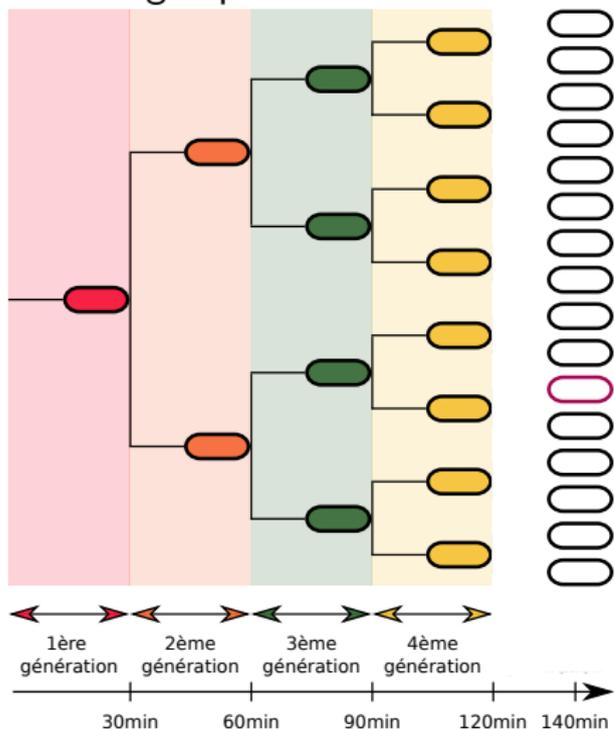
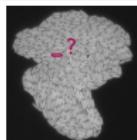
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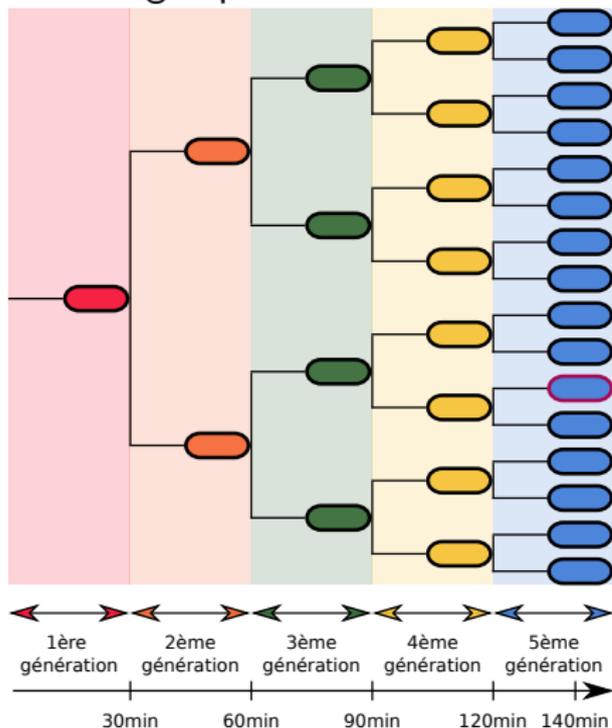
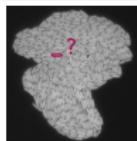
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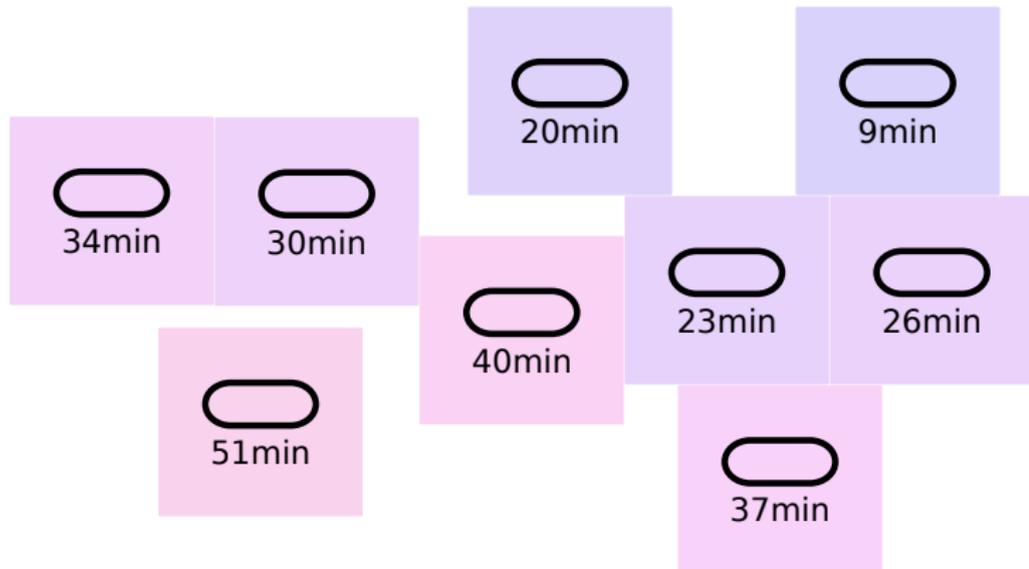


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Les cellules se divisent toutes les 30 minutes **en moyenne**.

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Exemple

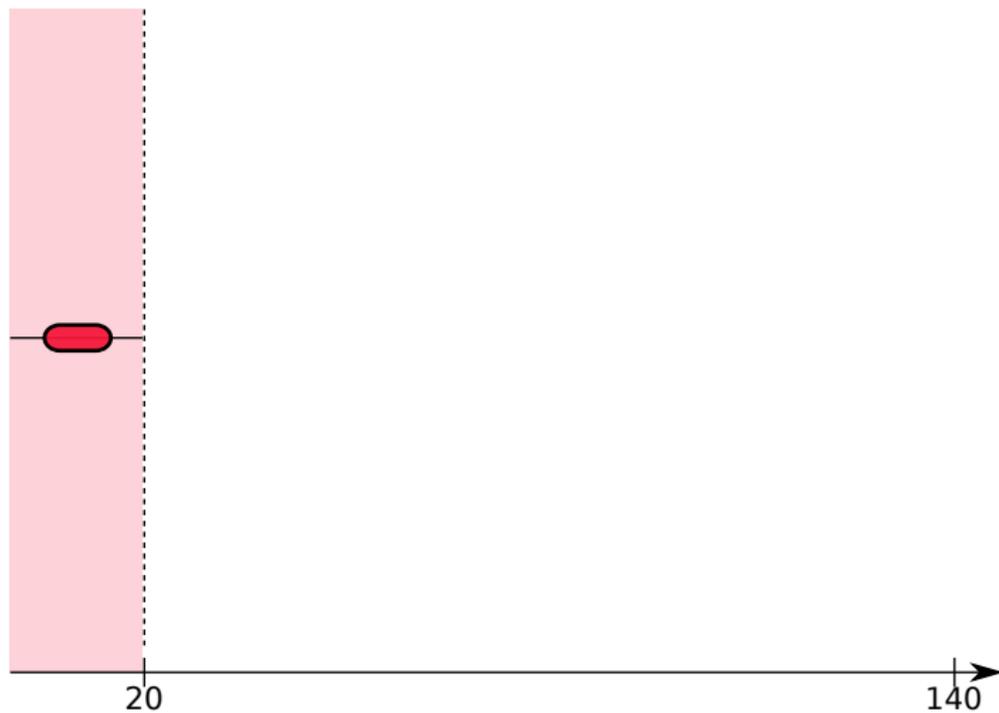
Cycles cellulaires de 20 min ou 40 min.



140

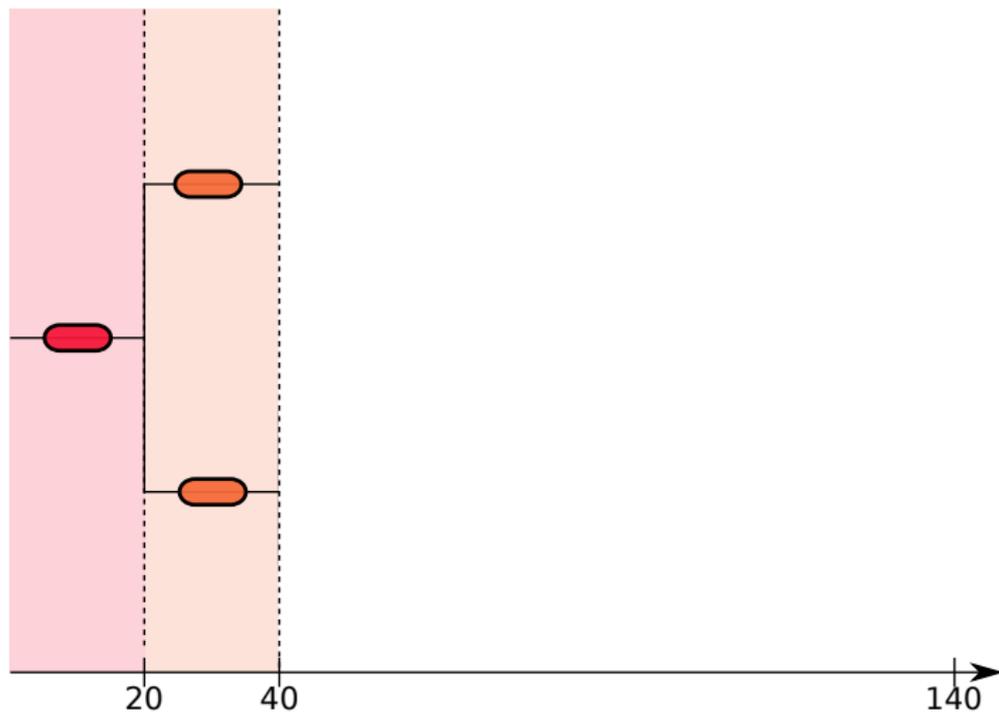
Exemple

Cycles cellulaires de 20 min ou 40 min.



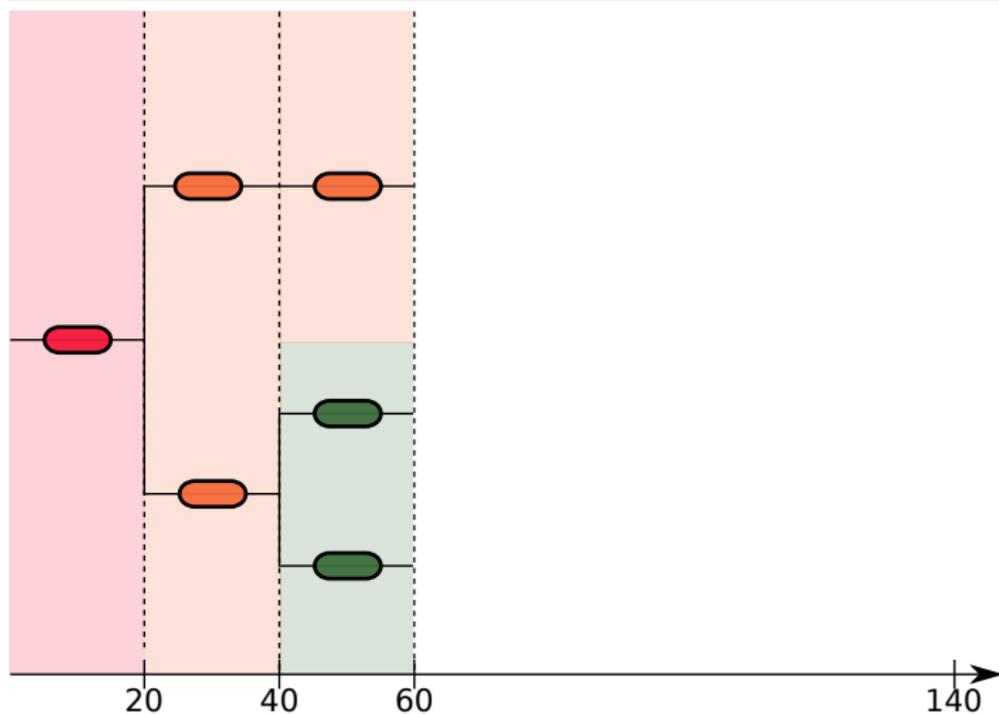
Exemple

Cycles cellulaires de 20 min ou 40 min.



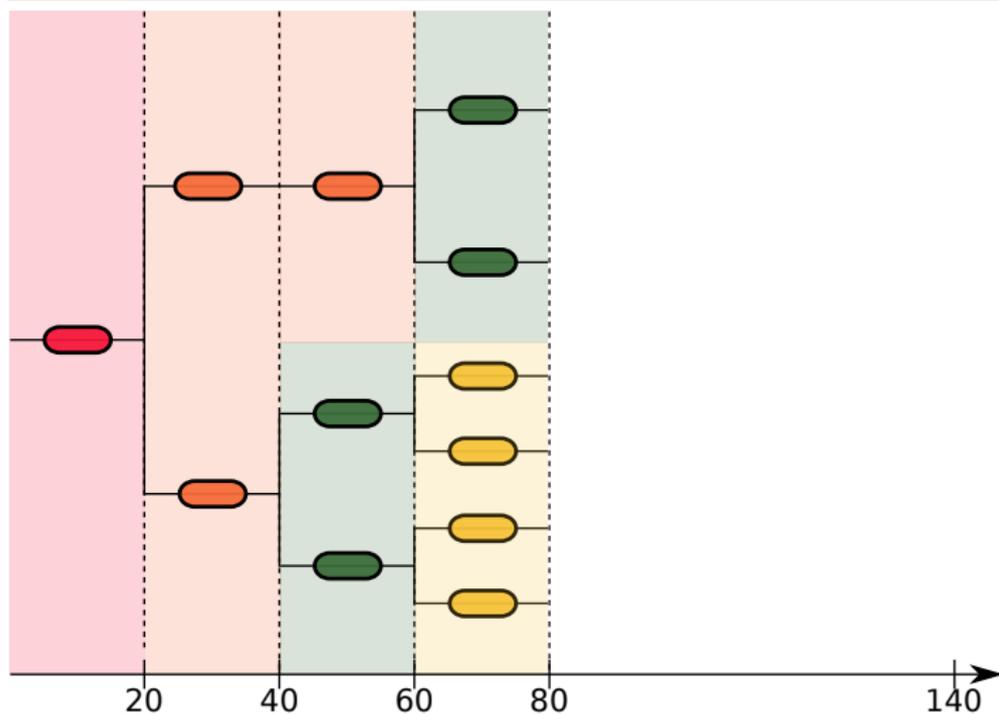
Exemple

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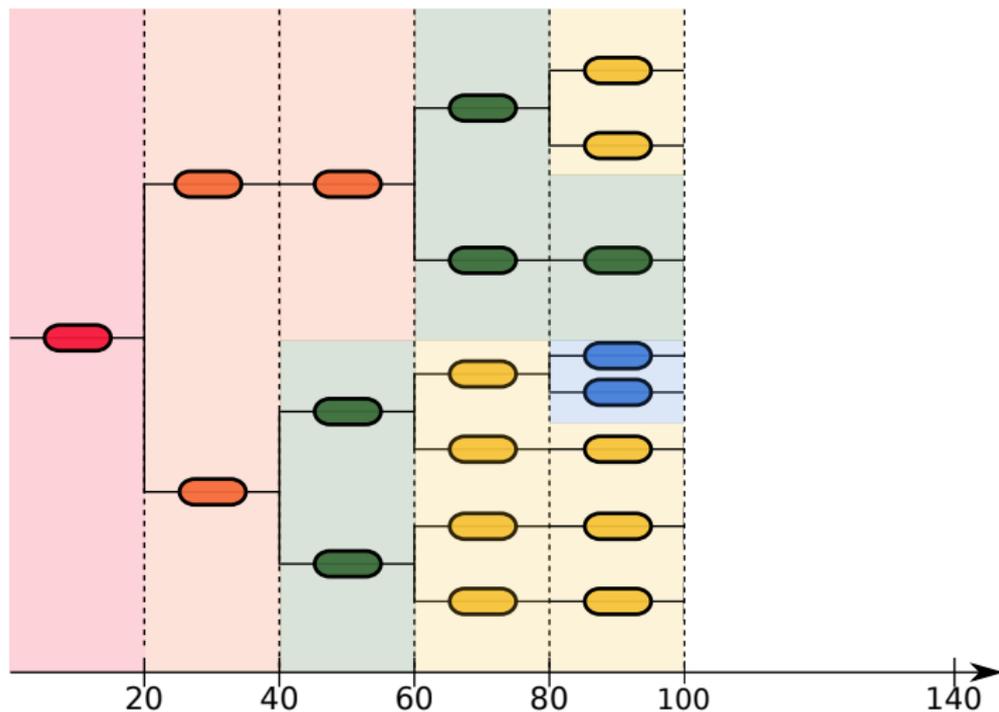
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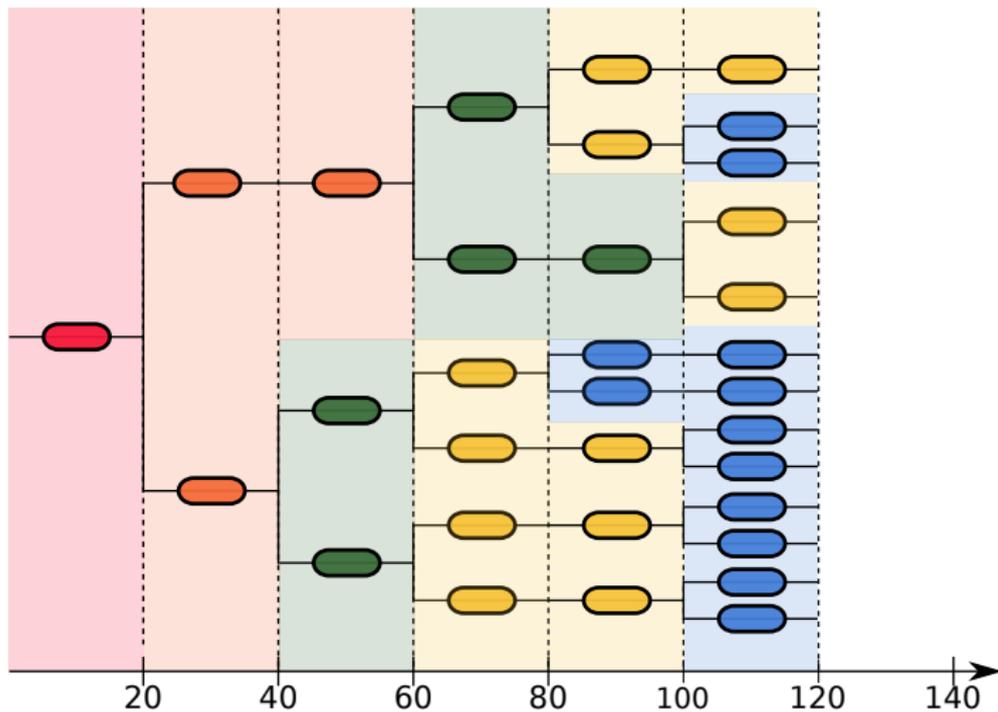
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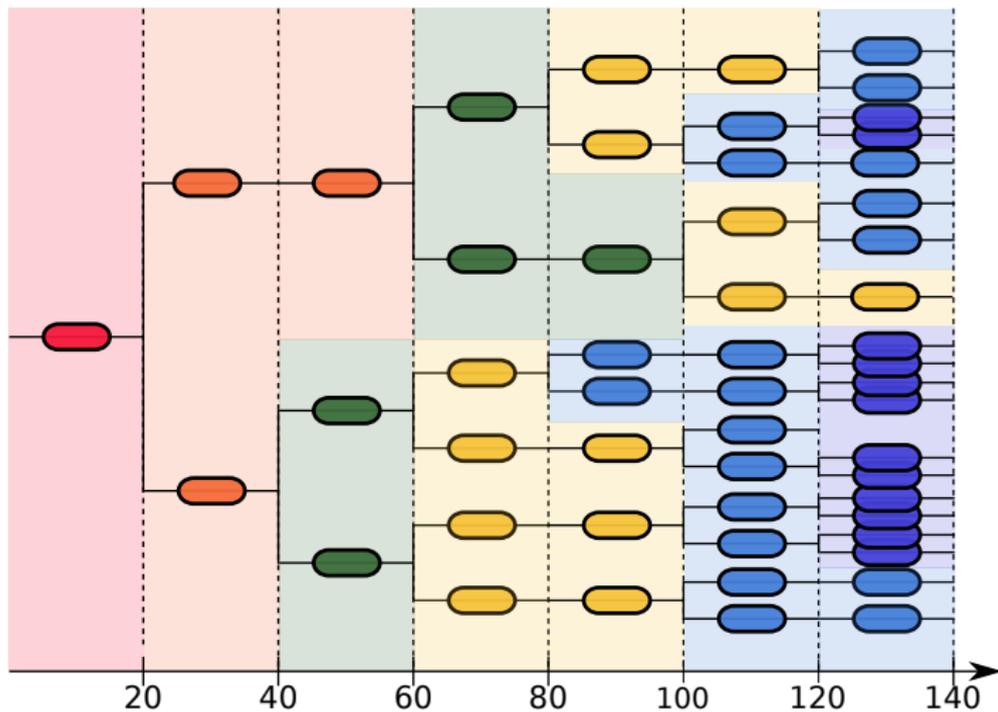
Exemple

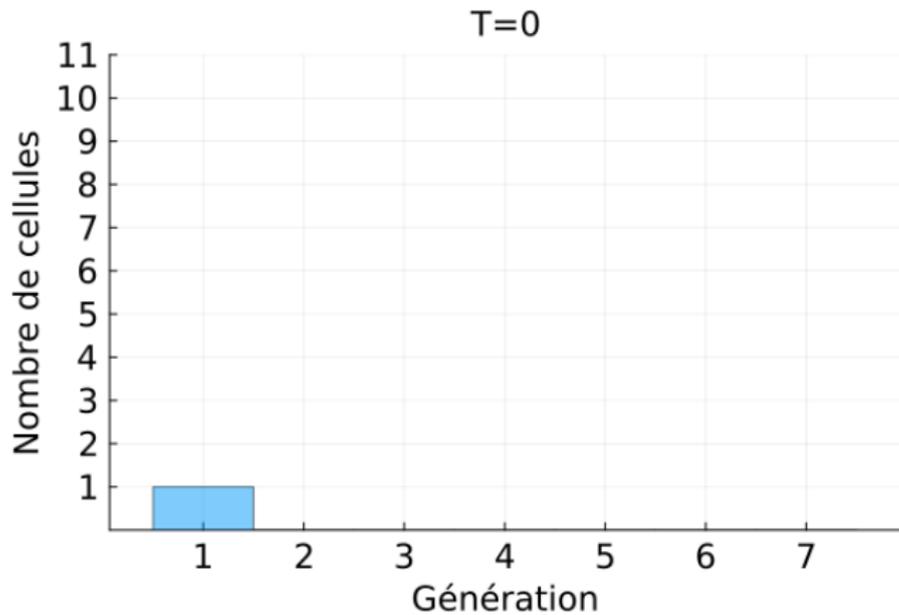
Cycles cellulaires de 20 min ou 40 min.

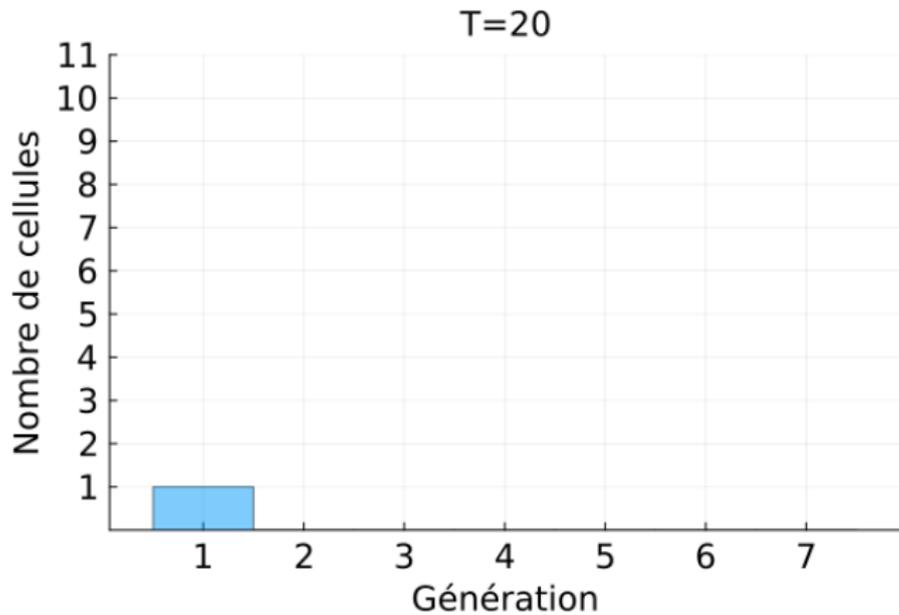


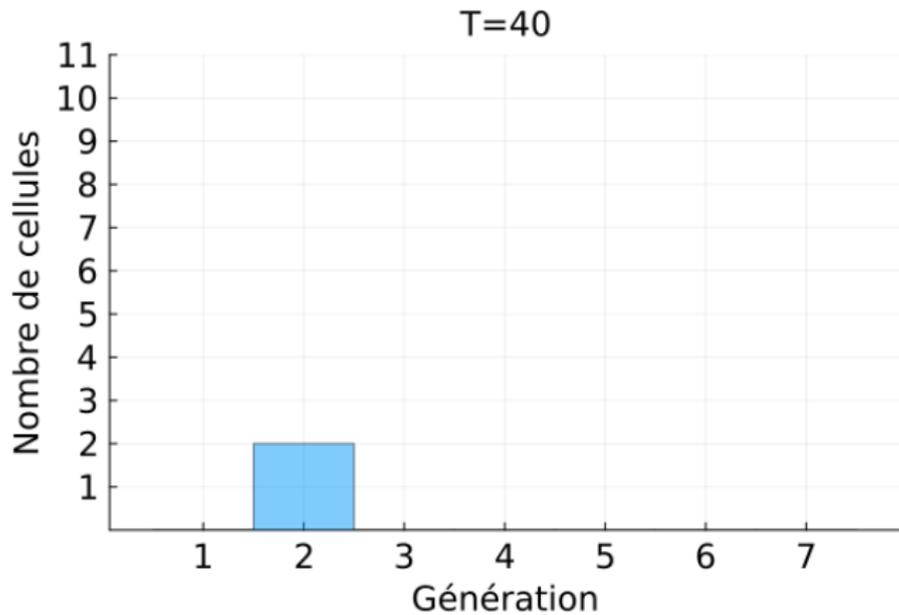
Exemple

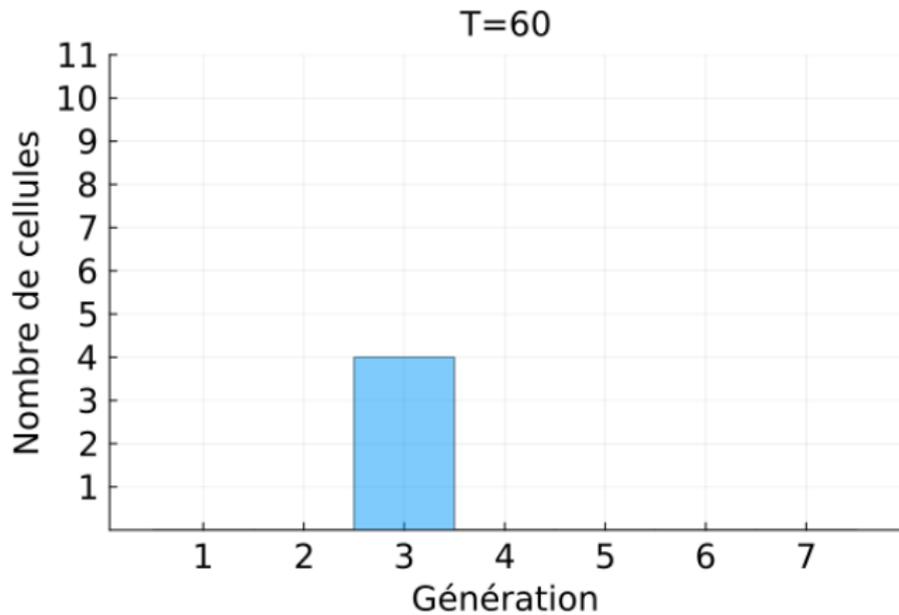
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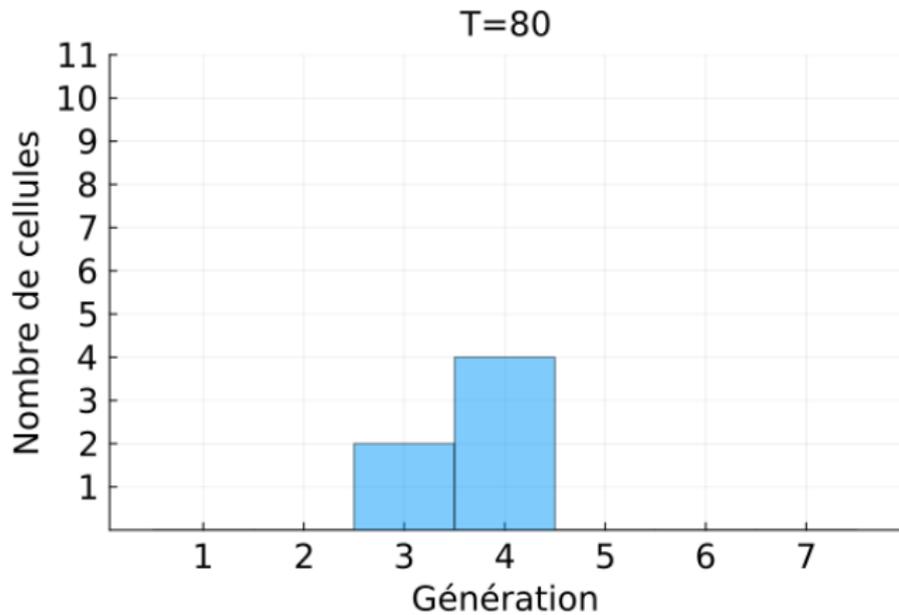


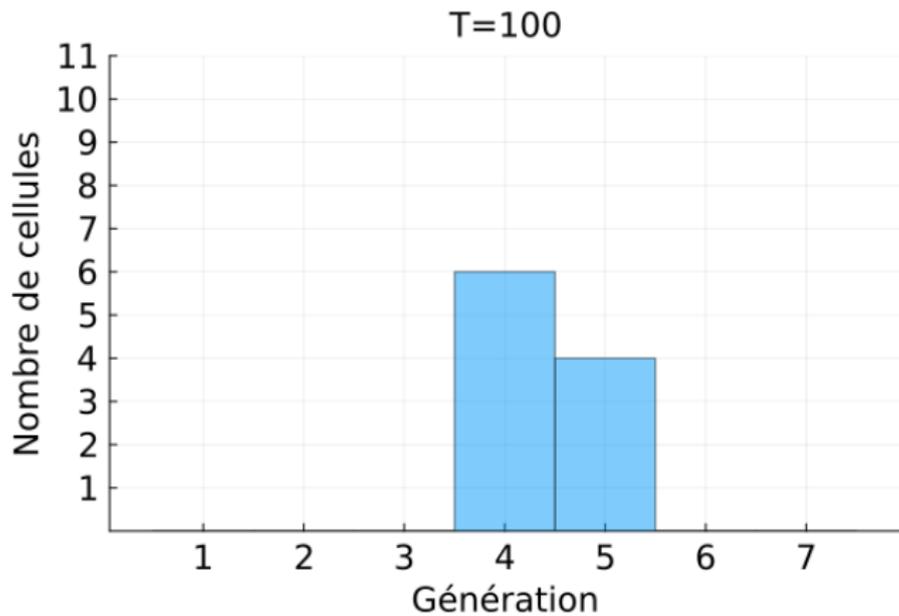


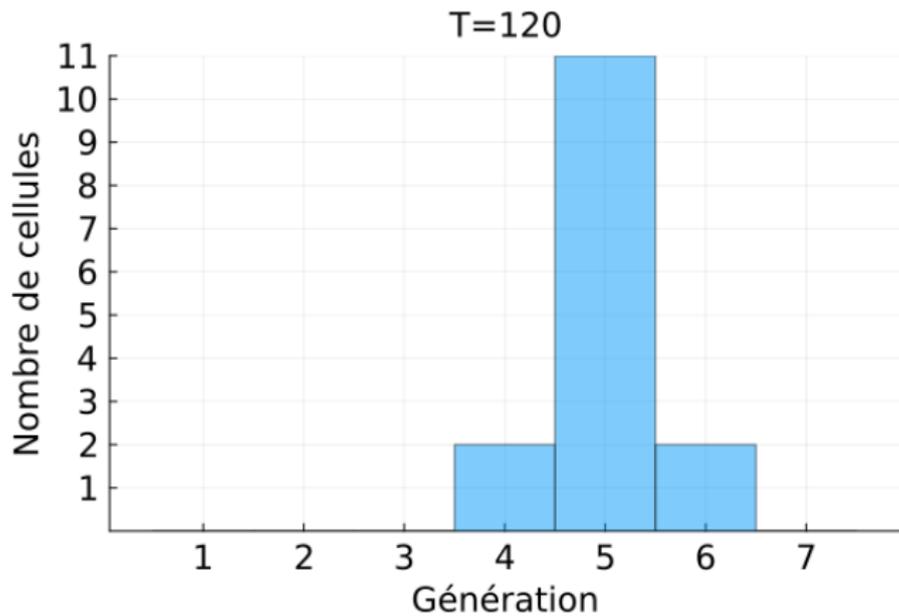


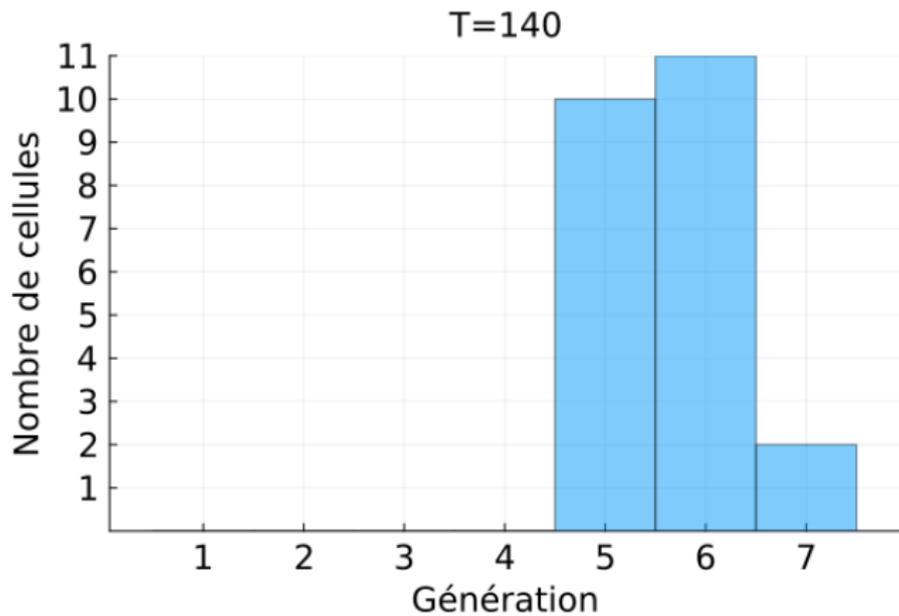


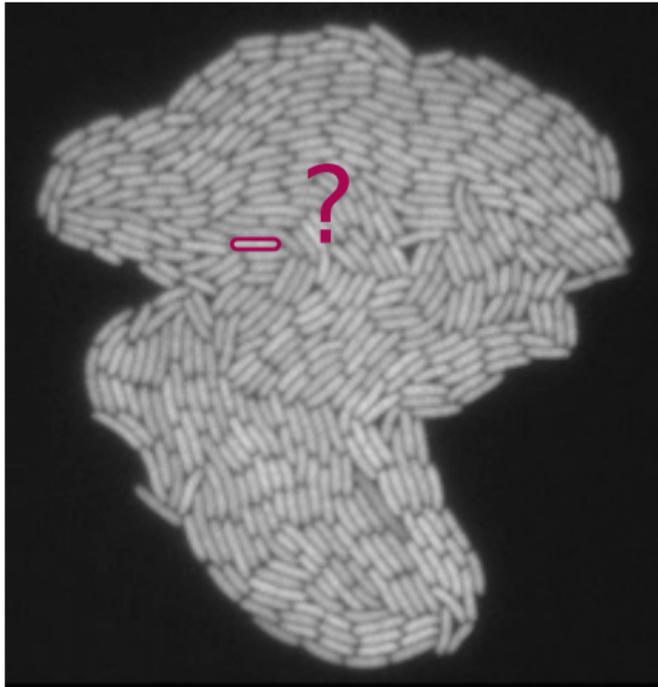












Durée du cycle cellulaire

- ◆ 20 min avec probabilité 0.5,
- ◆ 40 min avec probabilité 0.5.

Idée

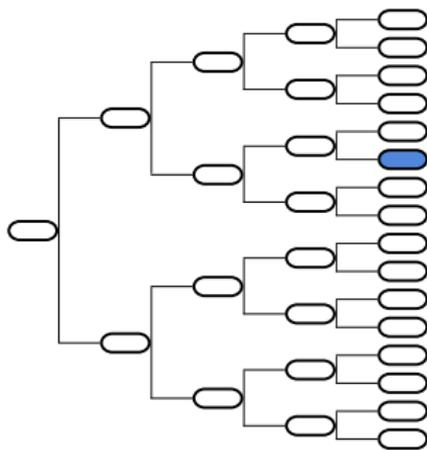
Modèle le long d'une lignée

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- ◆ 40 min avec probabilité 0.5.

Idée

Modèle le long d'une lignée

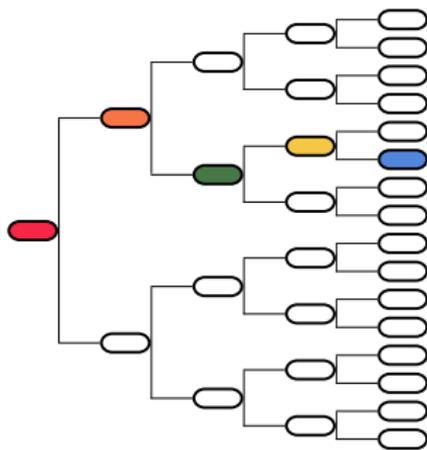


Durée du cycle cellulaire

- ◆ 20 min avec probabilité 0.5,
- ◆ 40 min avec probabilité 0.5.

Idée

Modèle le long d'une lignée

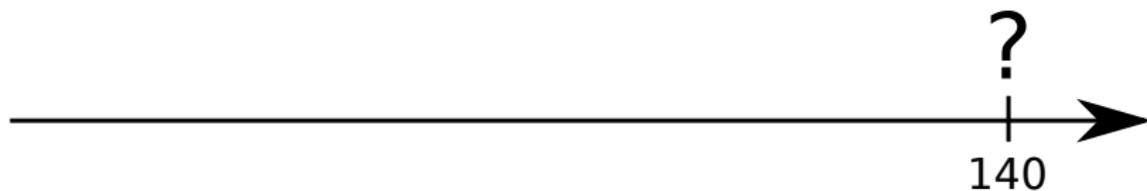


Exemple

Cycles cellulaires de 20 min ou 40 min.

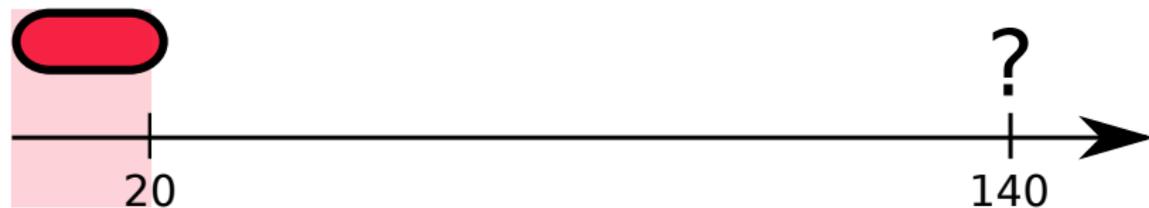
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Cycles cellulaires de 20 min ou 40 min.



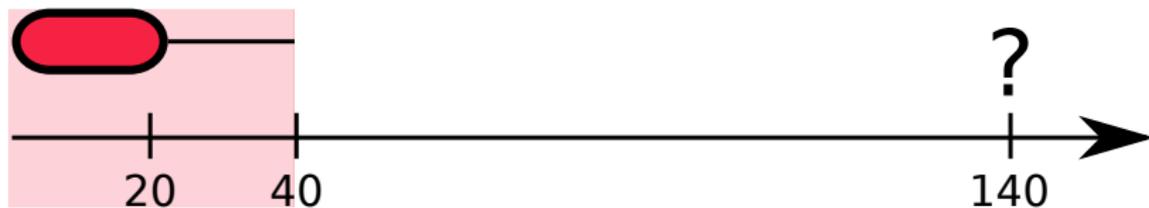
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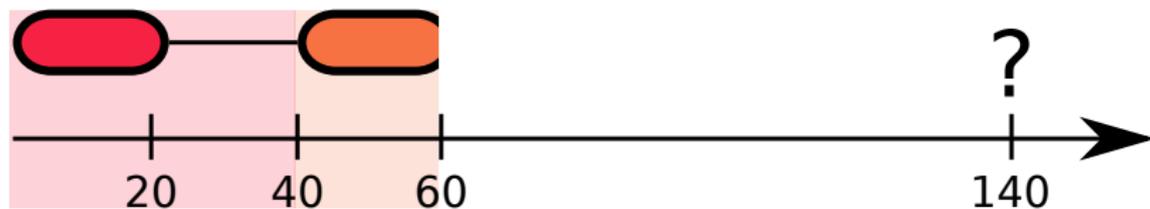
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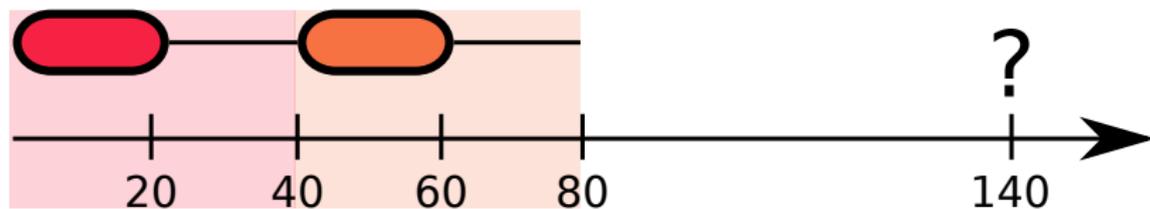
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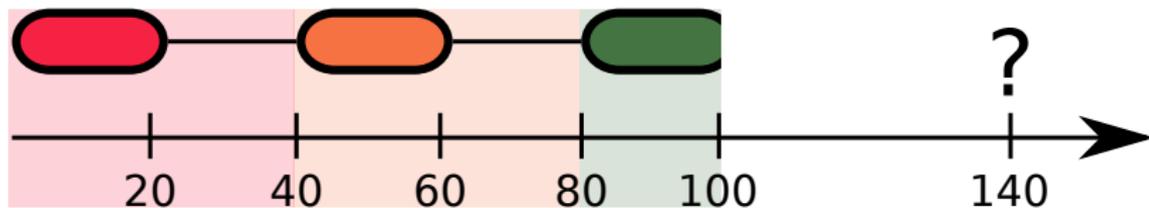
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Cycles cellulaires de 20 min ou 40 min.



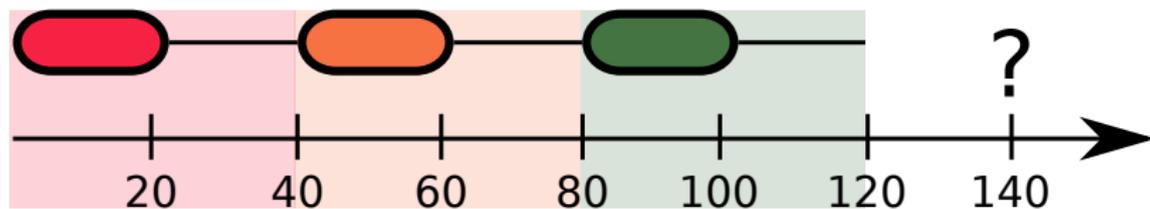
Exemple

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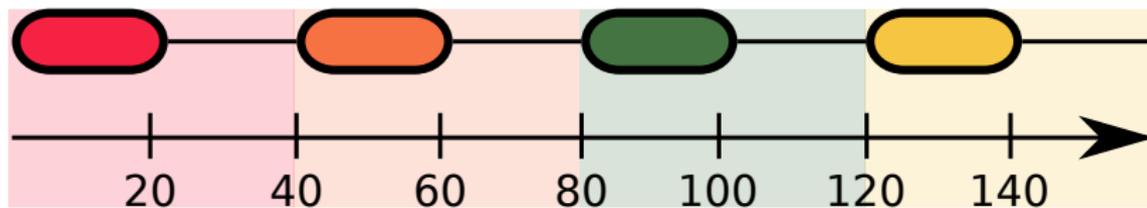
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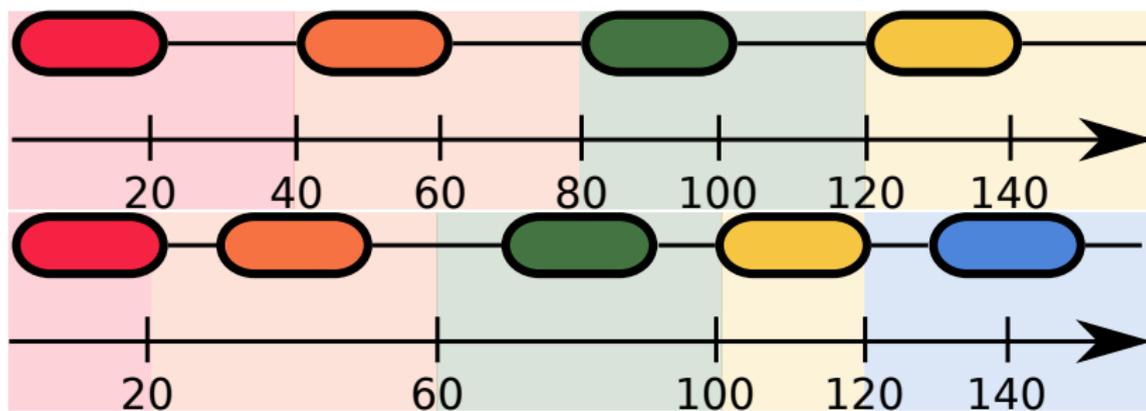
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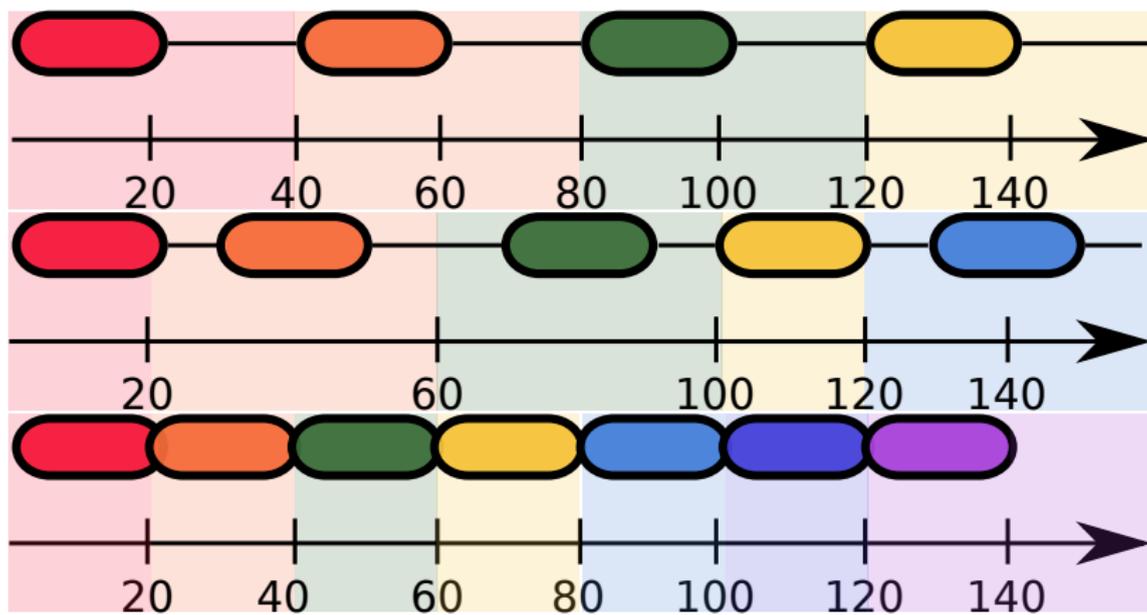
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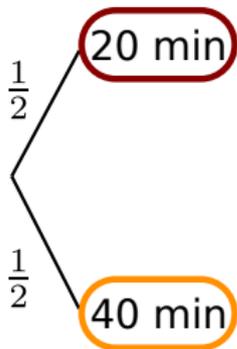


Exemple

Cycles cellulaires de 20 min ou 40 min.

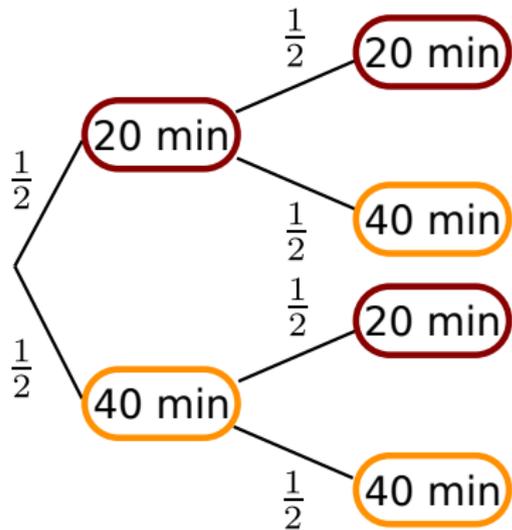


À $T = 30$,



$$\mathbb{P}(G = 1) = 1/2, \mathbb{P}(G = 2) = 1/2$$

À $T = 50$,



$$\mathbb{P}(G = 1) = 0, \quad \mathbb{P}(G = 2) = 3/4, \quad \mathbb{P}(G = 3) = 1/4$$

À $T = 70$,



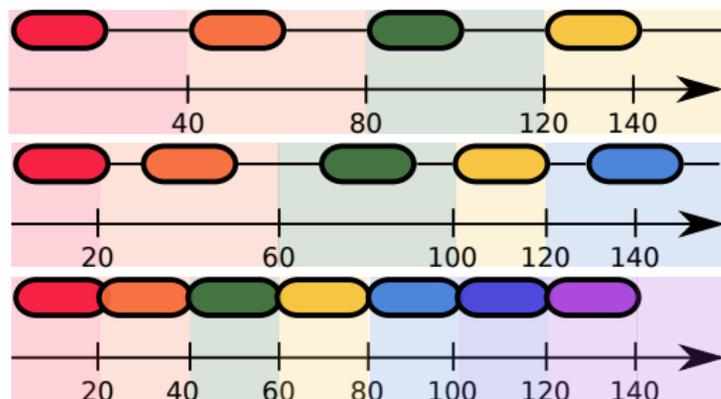
$$\mathbb{P}(G = 1) = 0, \quad \mathbb{P}(G = 2) = 1/4, \quad \mathbb{P}(G = 3) = 5/8, \quad \mathbb{P}(G = 4) = 1/8$$

Durée du cycle cellulaire

- ◆ 20 min avec probabilité 0.5,
- ◆ 40 min avec probabilité 0.5.

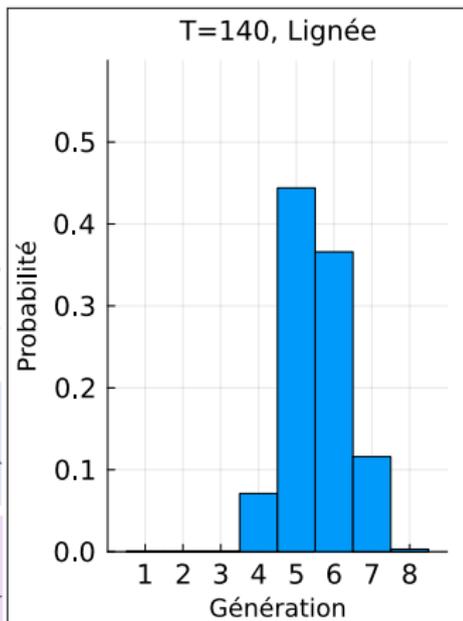
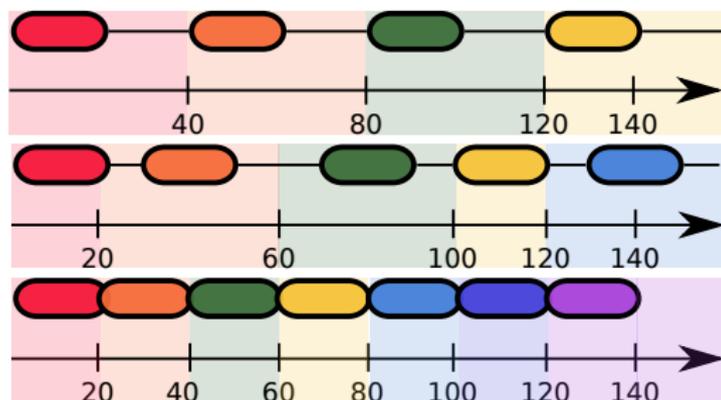
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Durée du cycle cellulaire

- ◆ 20 min avec probabilité 0.5,
- ◆ 40 min avec probabilité 0.5.



$$\mathbb{P}(G = 4) = 8/128, \quad \mathbb{P}(G = 5) = 56/128, \quad \mathbb{P}(G = 6) = 50/128,$$

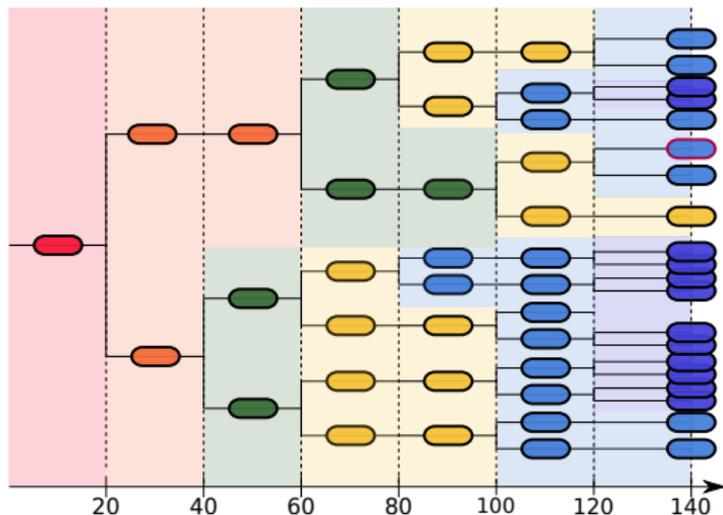
$$\mathbb{P}(G = 7) = 13/128, \quad \mathbb{P}(G = 8) = 1/128. \quad \mathbb{E}[G] = 5.6$$

Durée du cycle cellulaire

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- ◆ 40 min avec probabilité 0.5.

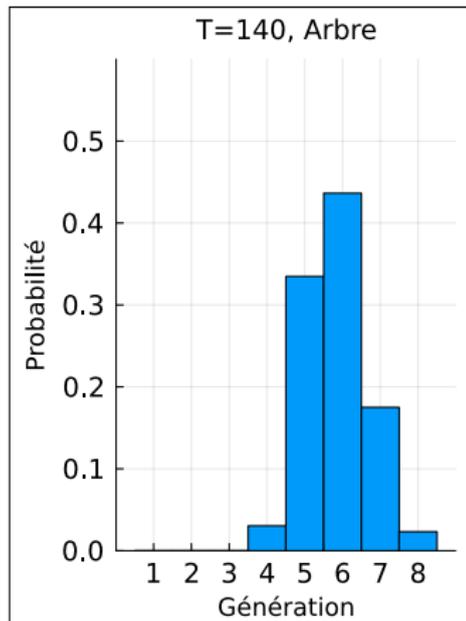
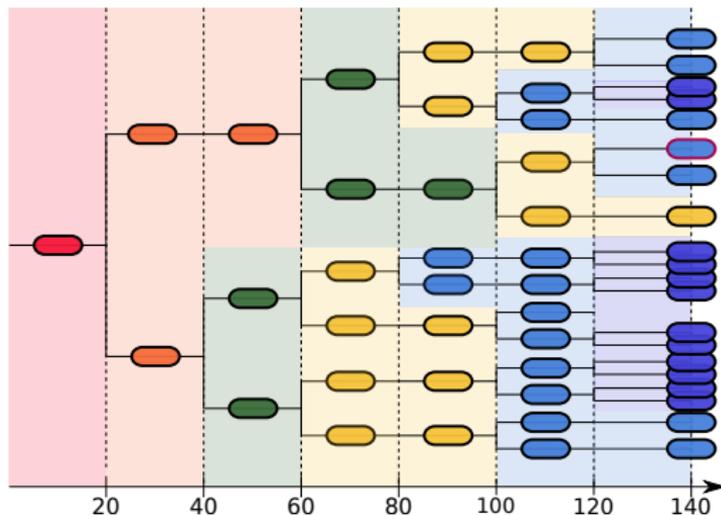
Durée du cycle cellulaire

- ◆ 20 min avec probabilité 0.5,
- ◆ 40 min avec probabilité 0.5.



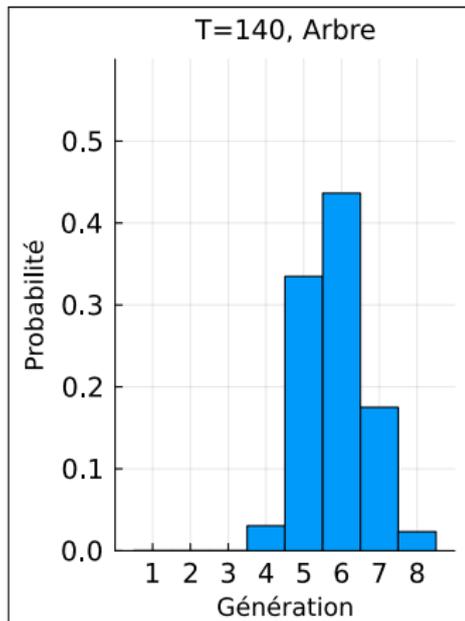
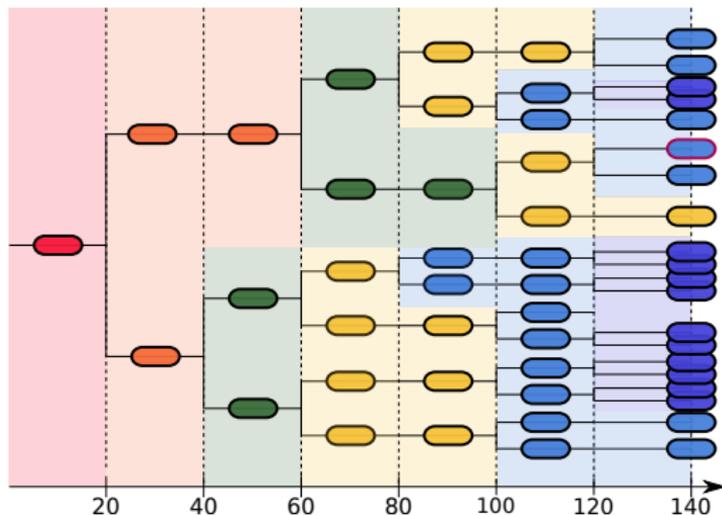
Durée du cycle cellulaire

- ◆ 20 min avec probabilité 0.5,
- ◆ 40 min avec probabilité 0.5.



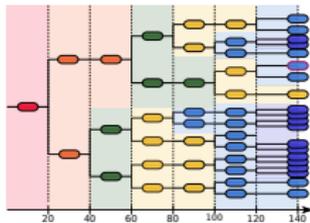
Durée du cycle cellulaire

- ◆ 20 min avec probabilité 0.5,
- ◆ 40 min avec probabilité 0.5.

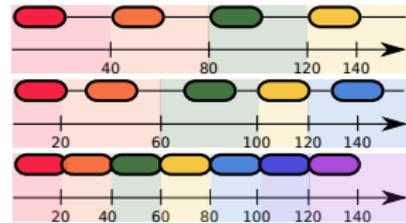
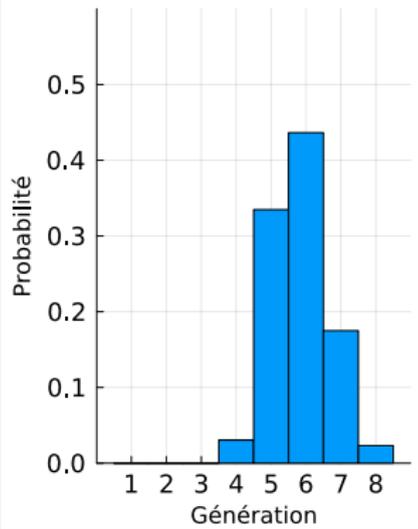


$$\mathbb{E}[G] = 5.9$$

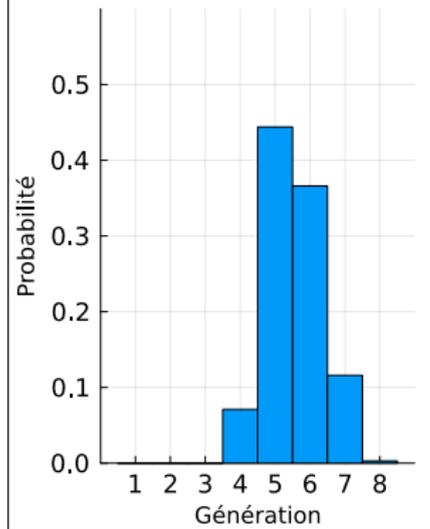
Comparaison



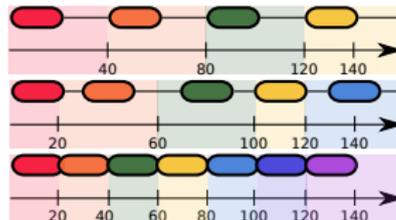
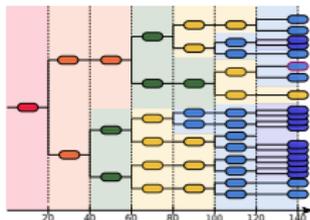
T=140, Arbre



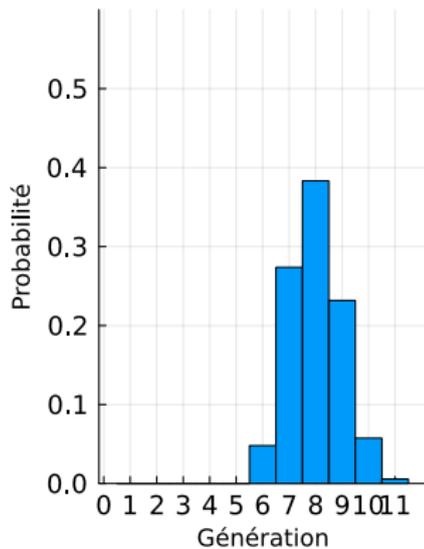
T=140, Lignée



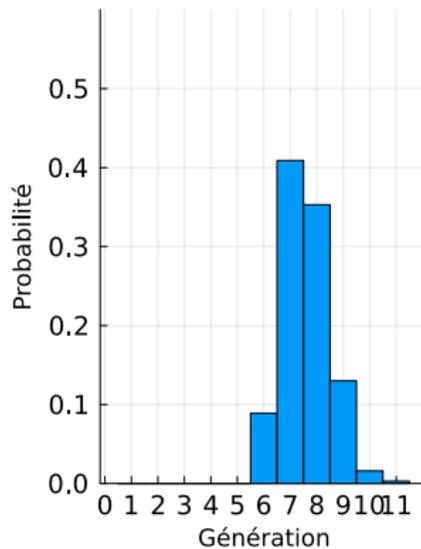
Comparaison en temps long

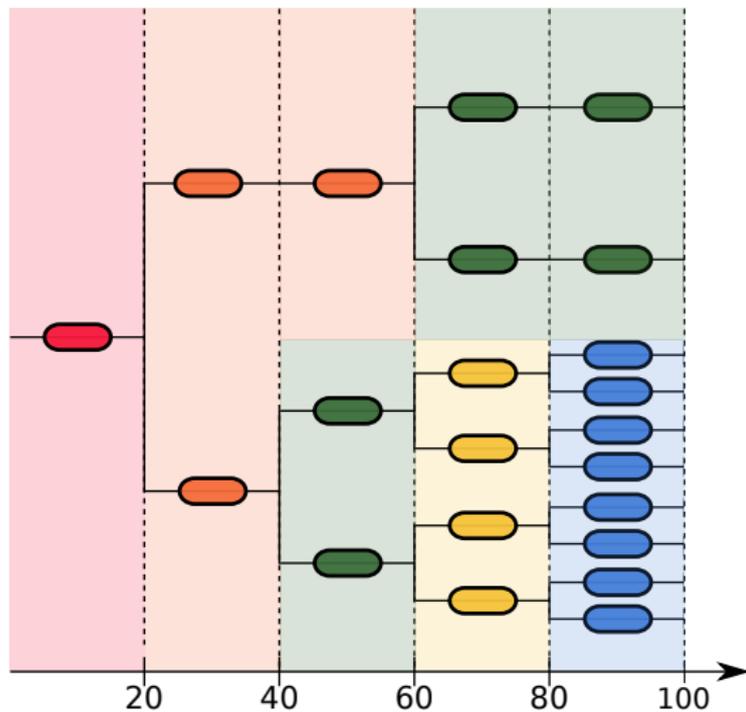


T=200, Arbre

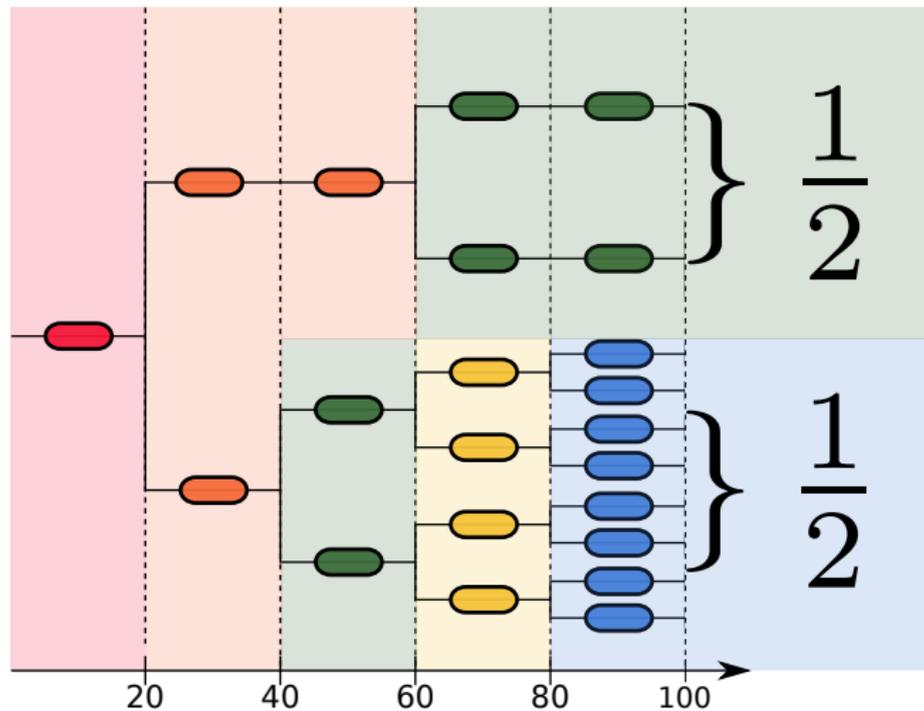


T=200, Lignée

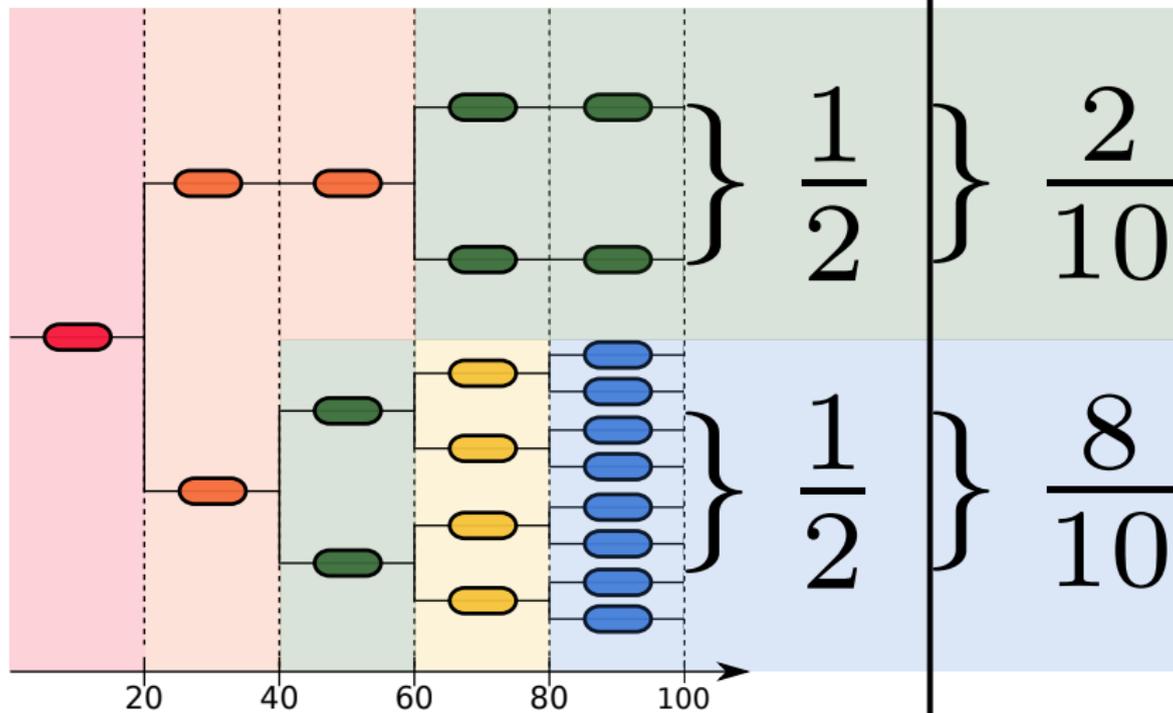




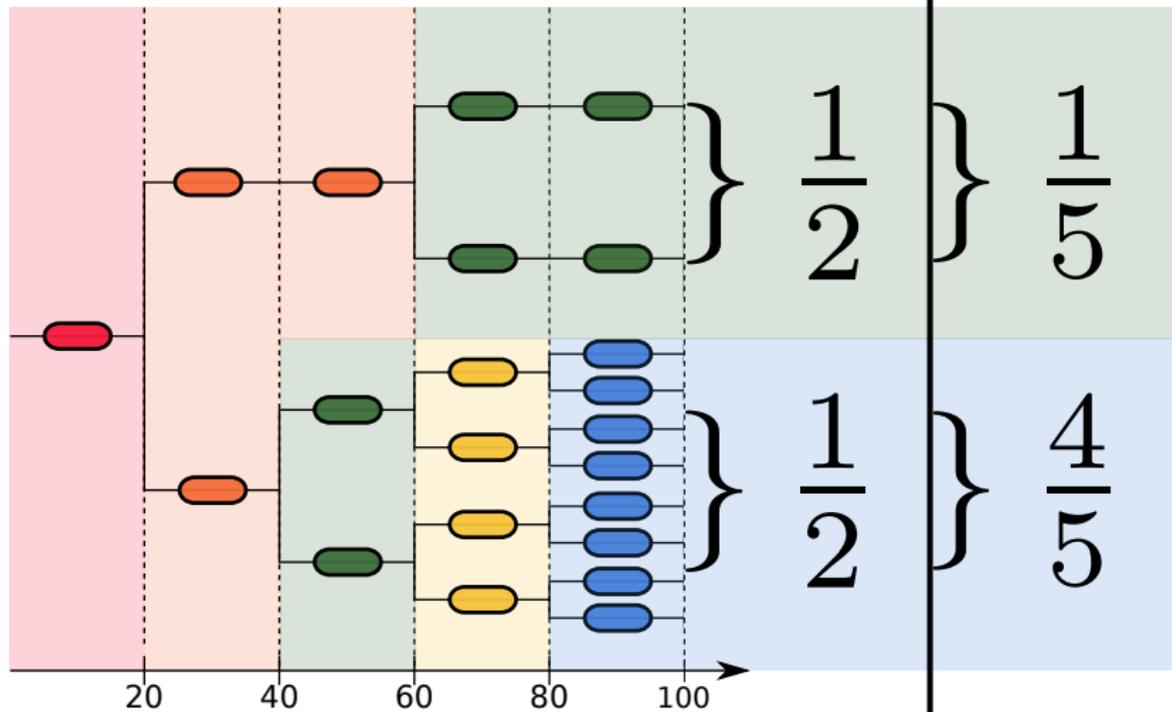
Lignée



Lignée | Arbre



Lignée | Arbre



Modèles mathématiques pour :

- ◆ Se substituer à des mesures fastidieuses,
- ◆ Comprendre des dynamiques complexes,
- ◆ Prévoir des comportements,
- ◆ etc.

Généralisations :

- ◆ N'importe quelle durée de vie,
- ◆ Mécanismes intracellulaires,
- ◆ Élongation,
- ◆ etc.

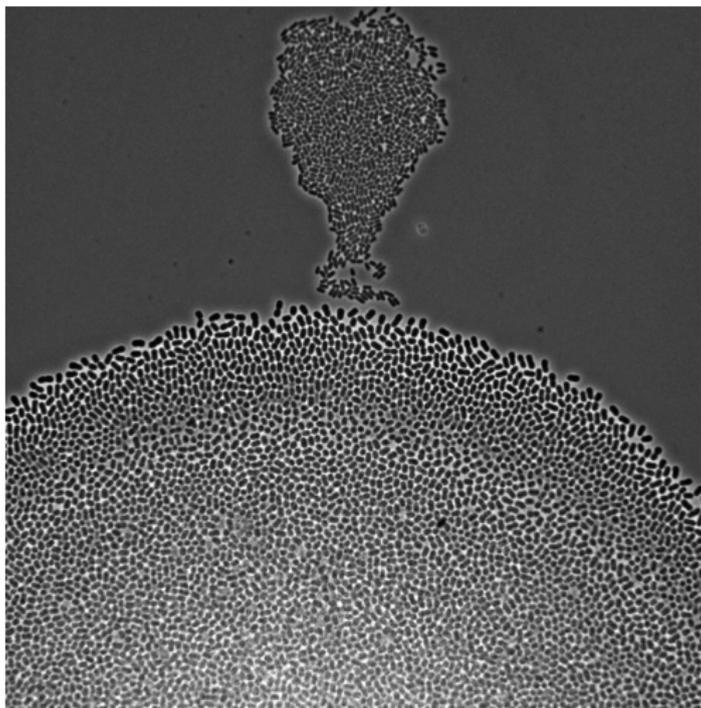


Figure: Montgolfière, par Antrea Pavlou

Merci pour votre attention !