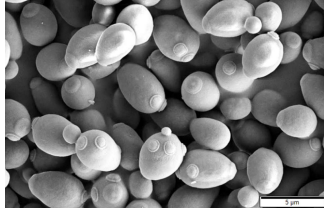
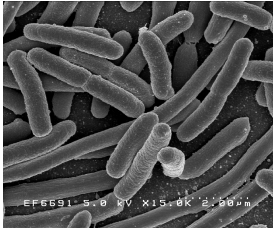


Inria

Modélisation mathématique des populations de cellules

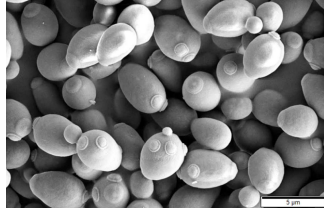
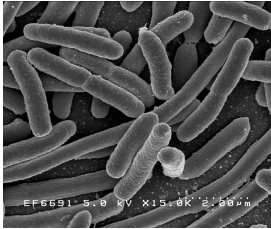
A. Marguet

◆ Différents organismes



◆ Bactéries : *Escherichia Coli* ◆ Levures : *Saccharomyces cerevisiae*

◆ Différents organismes



◆ Bactéries : *Escherichia Coli* ◆ Levures : *Saccharomyces cerevisiae*

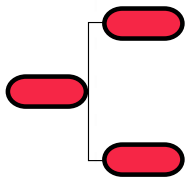
◆ Différentes applications

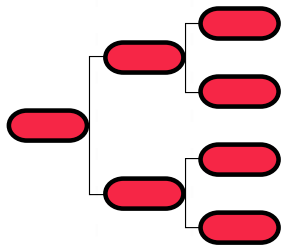


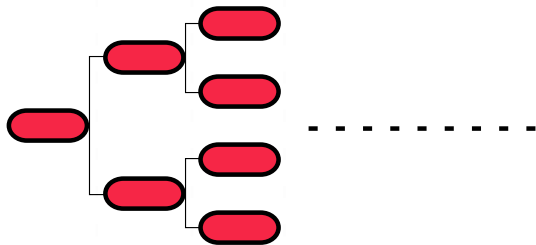
01

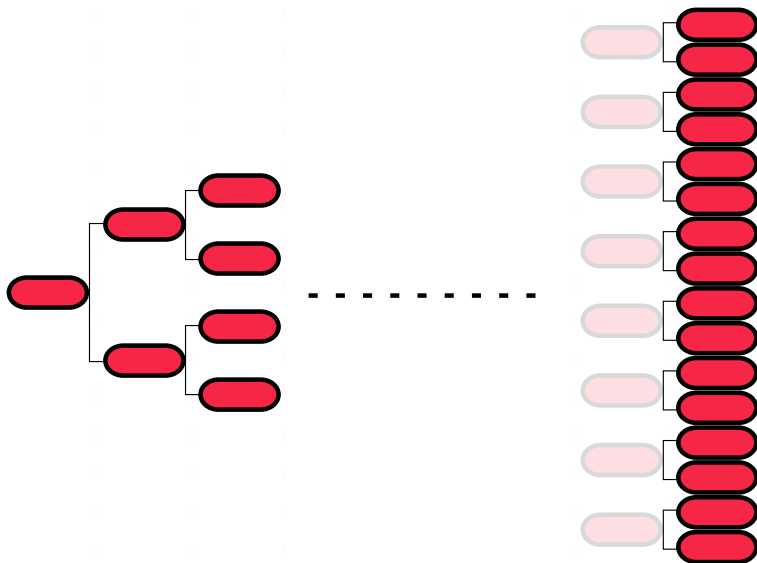
La croissance











On note X_n le nombre de cellules à la génération n .

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$$X_1 =$$

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$$X_1 = 1$$

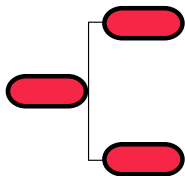
On note X_n le nombre de cellules à la génération n .



$$X_1 = 1$$

$$X_2 =$$

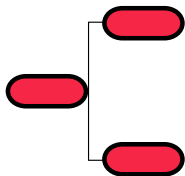
On note X_n le nombre de cellules à la génération n .



$$X_1 = 1$$

$$X_2 = 2$$

On note X_n le nombre de cellules à la génération n .

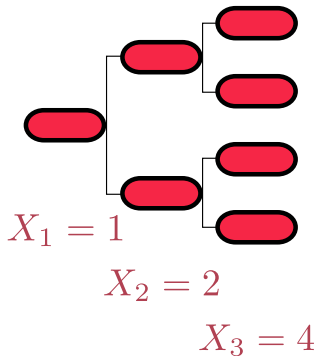


$$X_1 = 1$$

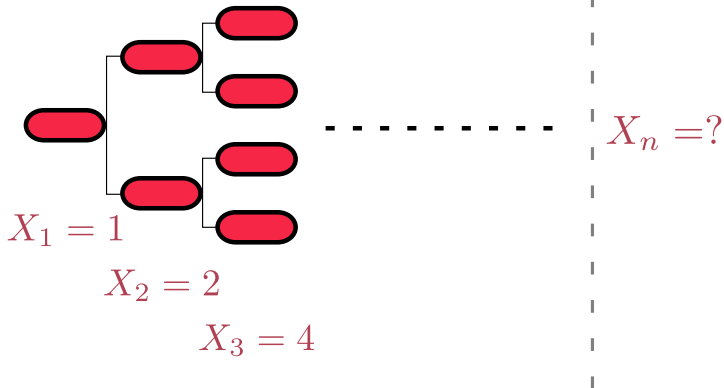
$$X_2 = 2$$

$$X_3 =$$

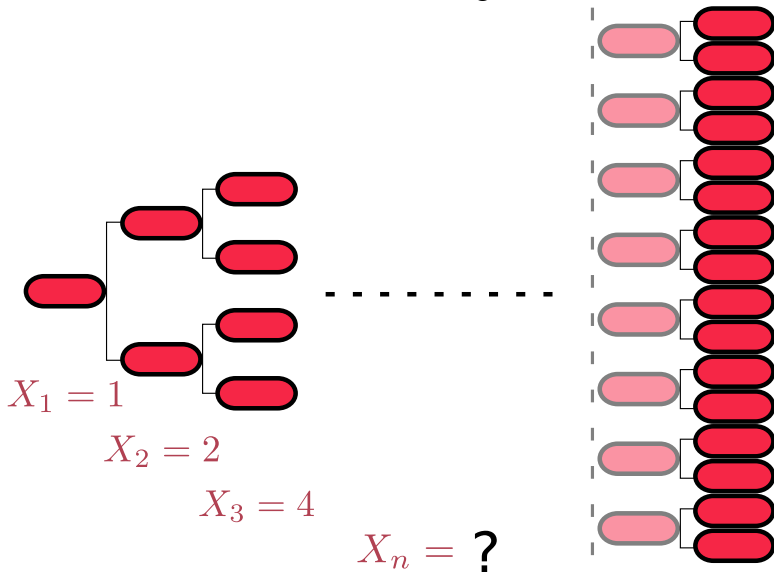
On note X_n le nombre de cellules à la génération n .



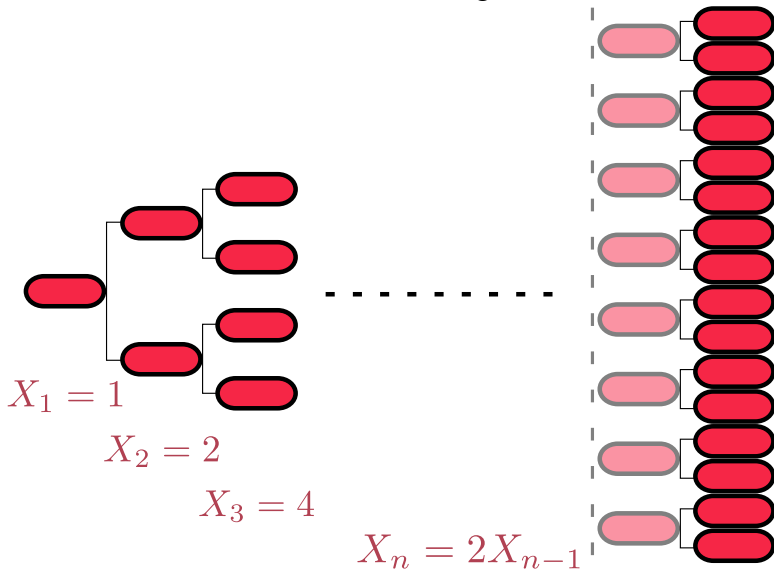
On note X_n le nombre de cellules à la génération n .

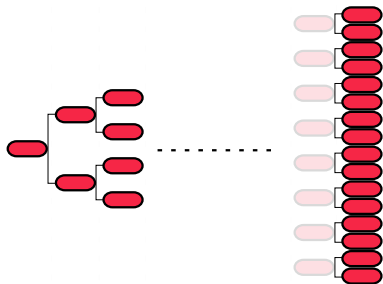


On note X_n le nombre de cellules à la génération n .

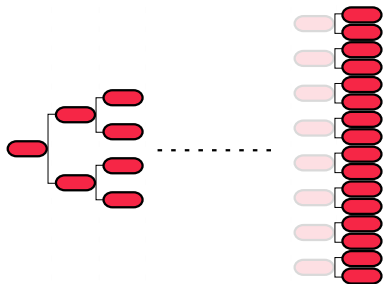


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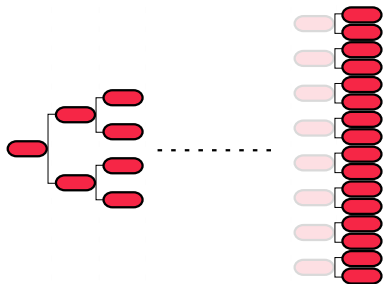




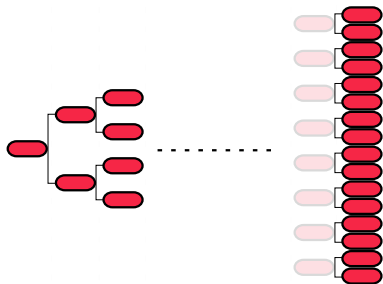
$$X_n = 2 \times X_{n-1}$$



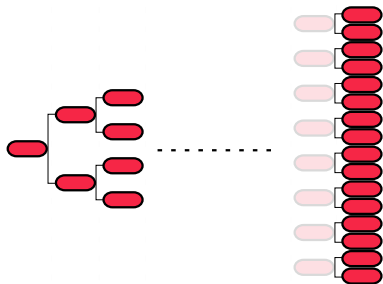
$$X_n = 2 \times X_{n-1} = 2 \times 2 \times X_{n-2}$$



$$X_n = 2 \times X_{n-1} = 2 \times 2 \times X_{n-2} = 2 \times 2 \times 2 \times X_{n-3}$$

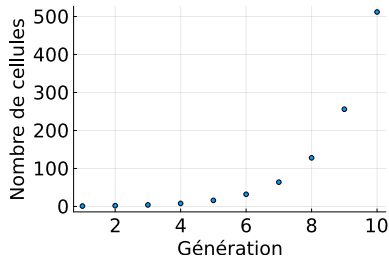
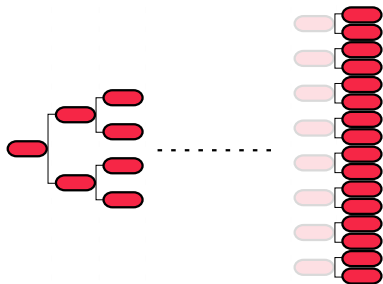


$$\begin{aligned}
 X_n &= 2 \times X_{n-1} = 2 \times 2 \times X_{n-2} = 2 \times 2 \times 2 \times X_{n-3} \\
 &= \underbrace{2 \times 2 \times \dots \times 2}_{n-1 \text{ fois}} \times X_1
 \end{aligned}$$



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$$X_n = 2^{n-1}$$



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 \end{aligned}$$

$$X_n = 2^{n-1}$$

02

Échantillonnage

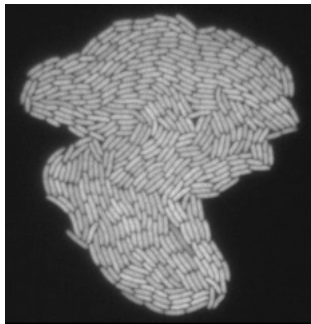
OPEN ACCESS PEER-REVIEWED

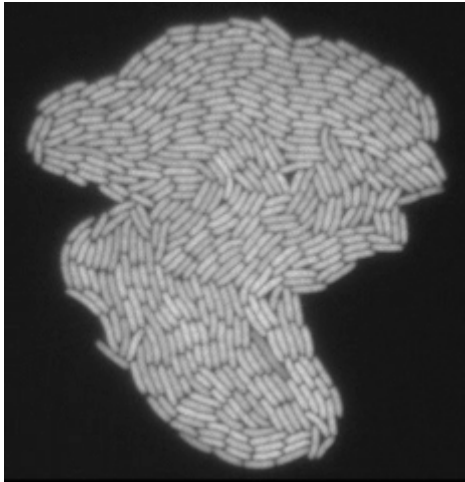
RESEARCH ARTICLE

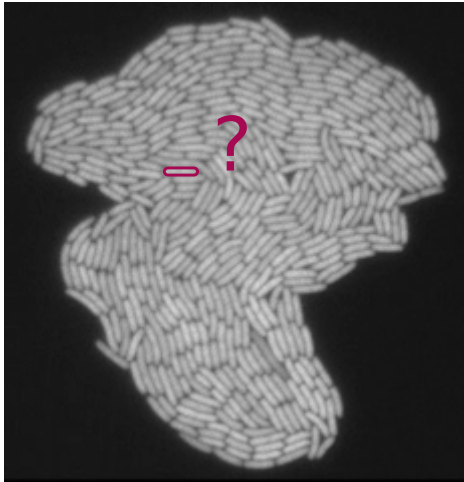
Aging and Death in an Organism That Reproduces by Morphologically Symmetric Division

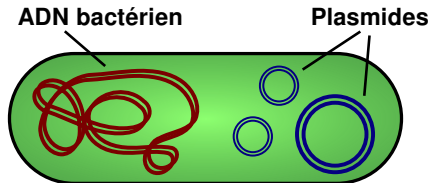
Eric J Stewart  Richard Madden, Gregory Paul, François Taddei

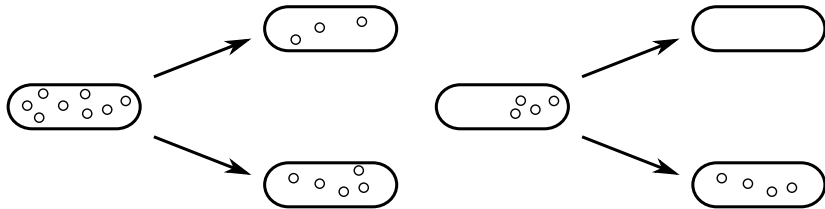
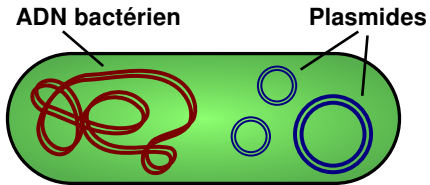
Published: February 1, 2005 • <https://doi.org/10.1371/journal.pbio.0030045>



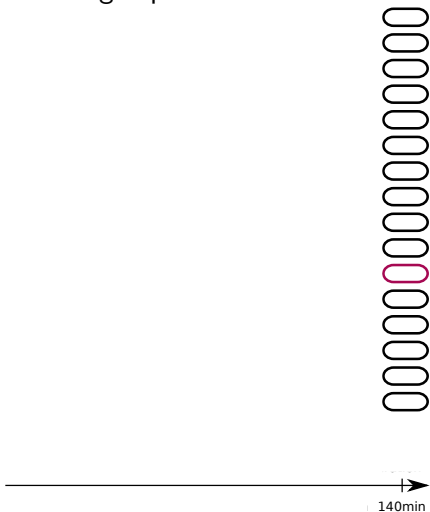




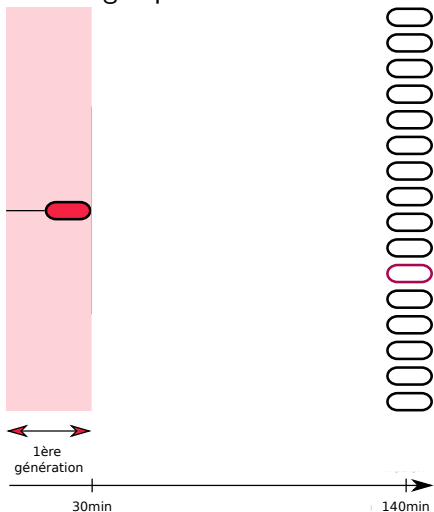
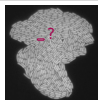




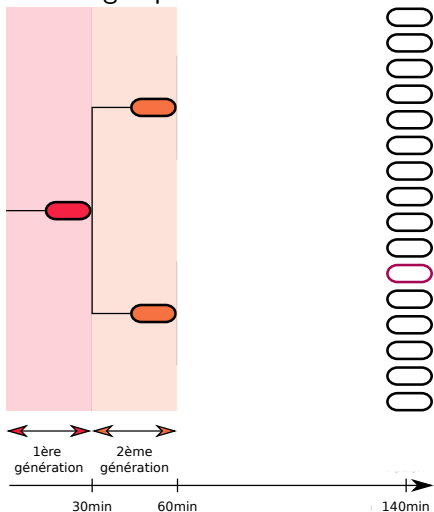
- ◆ Durée moyenne d'un cycle cellulaire : 30min.
- ◆ Échantillonnage après 140min.



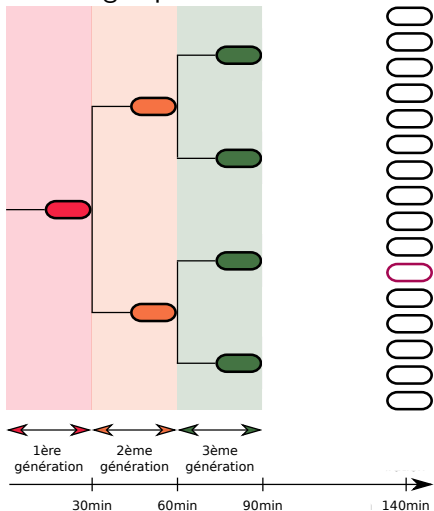
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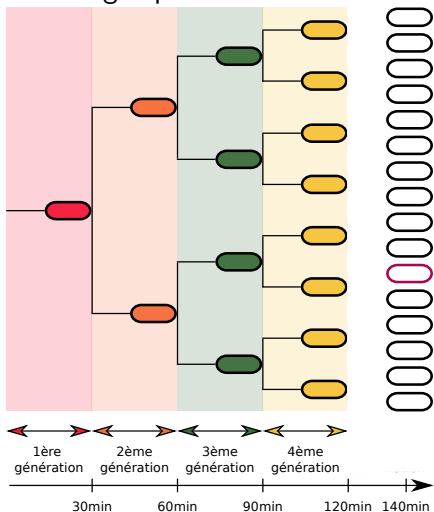
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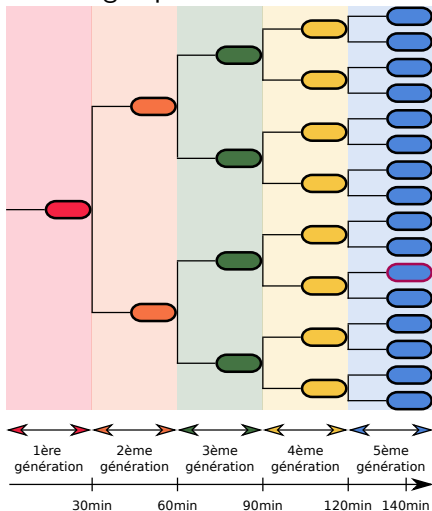
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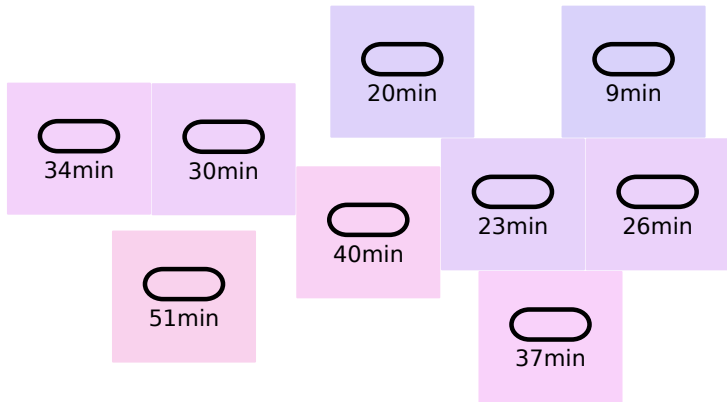


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Les cellules se divisent toutes les 30 minutes **en moyenne**.

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Exemple

Cycles cellulaires de 20 min ou 40 min.



140

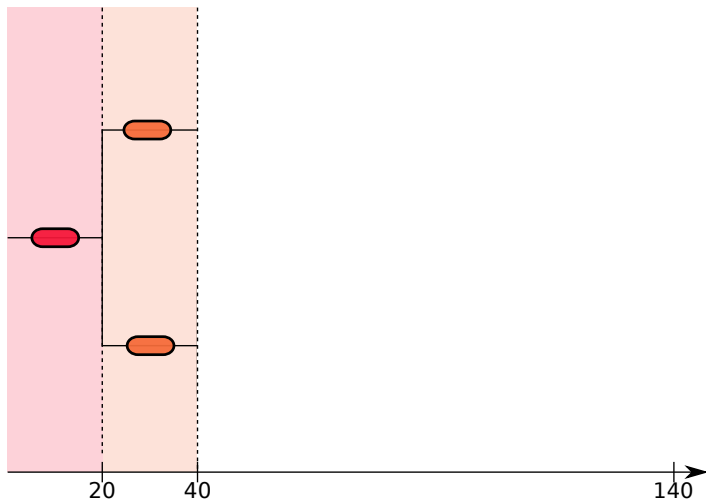
Exemple

Cycles cellulaires de 20 min ou 40 min.



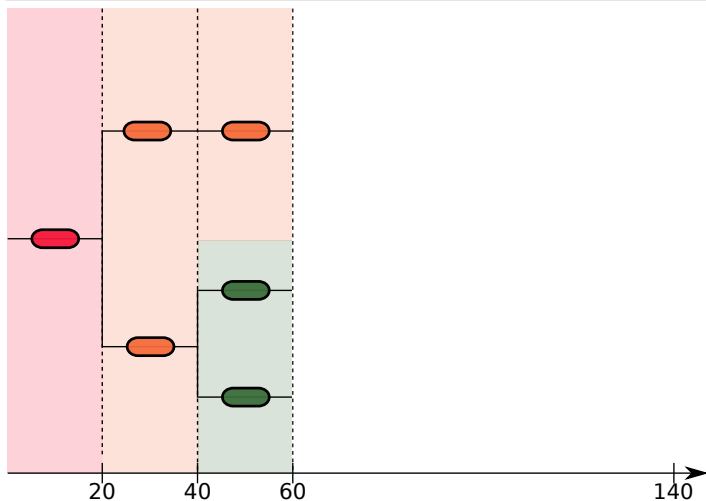
Exemple

Cycles cellulaires de 20 min ou 40 min.



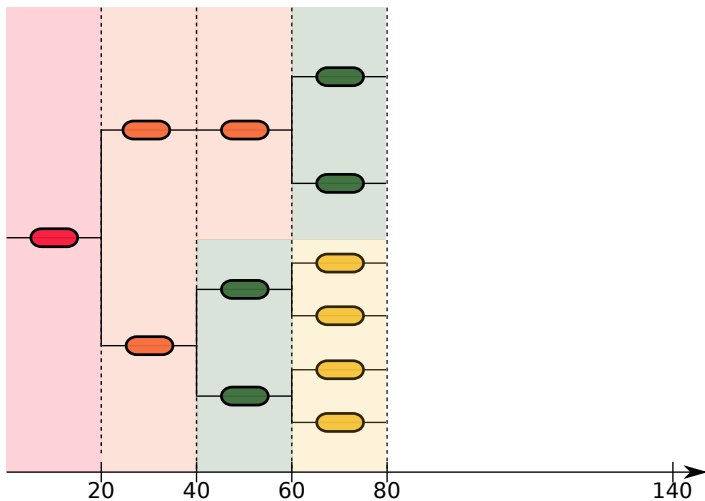
Exemple

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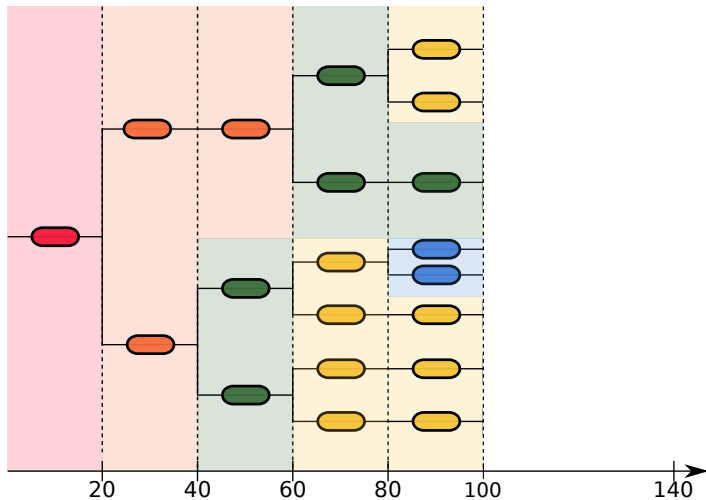
Exemple

Cycles cellulaires de 20 min ou 40 min.



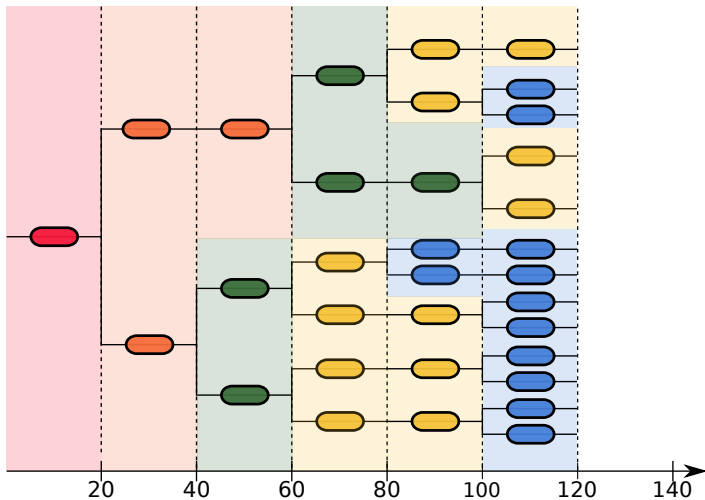
Exemple

Cycles cellulaires de 20 min ou 40 min.



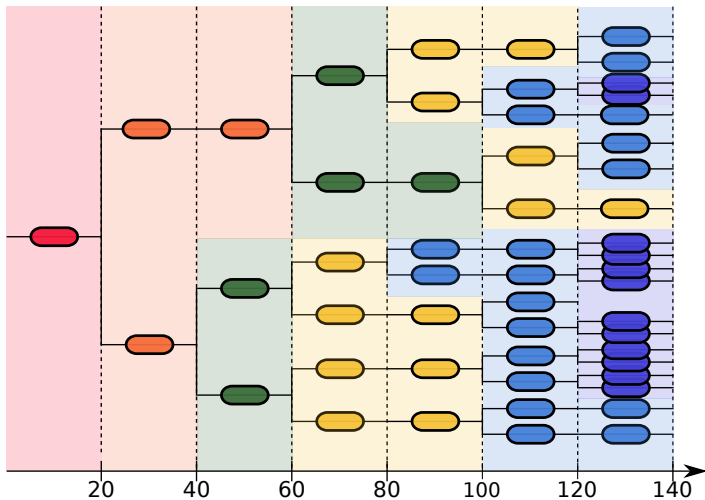
Exemple

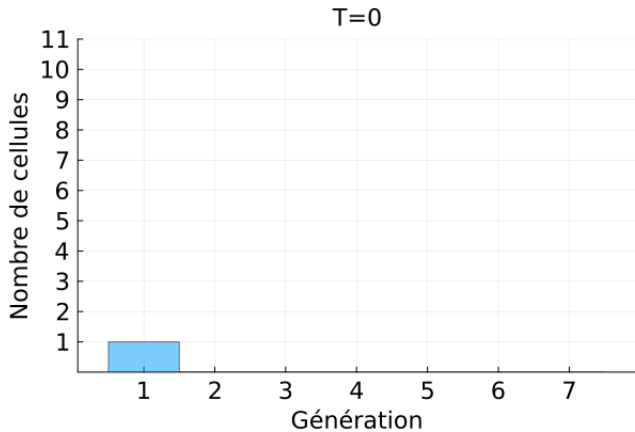
Cycles cellulaires de 20 min ou 40 min.

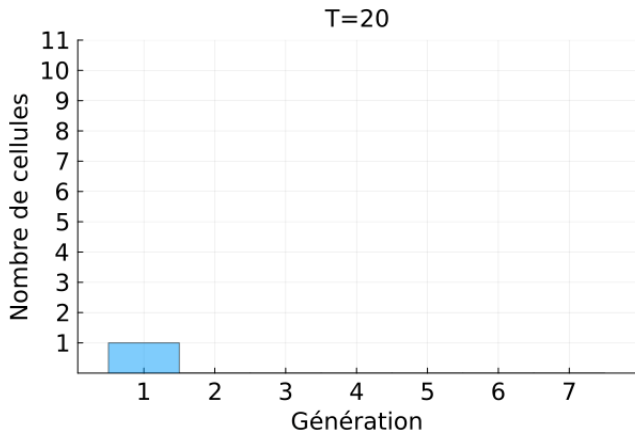


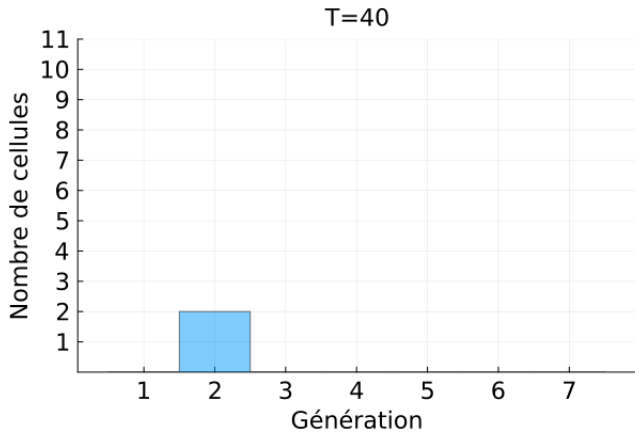
Exemple

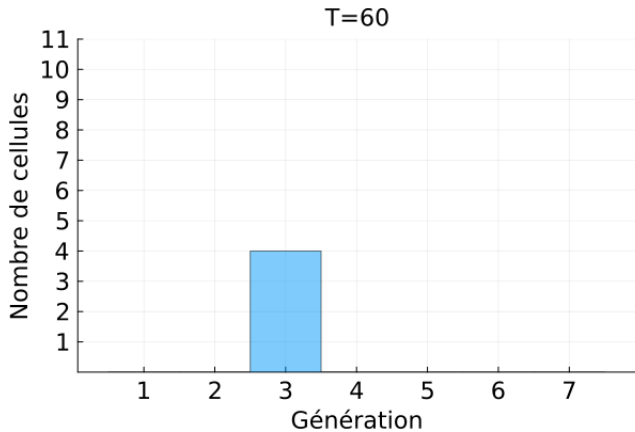
Cycles cellulaires de 20 min ou 40 min.

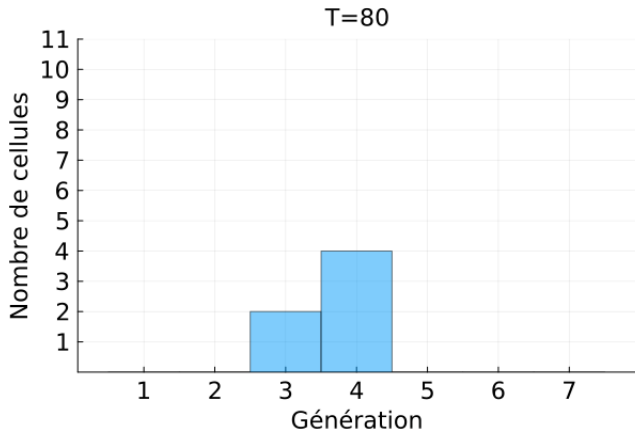


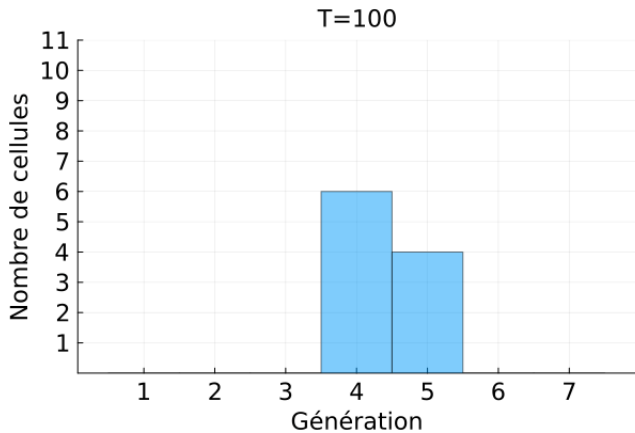


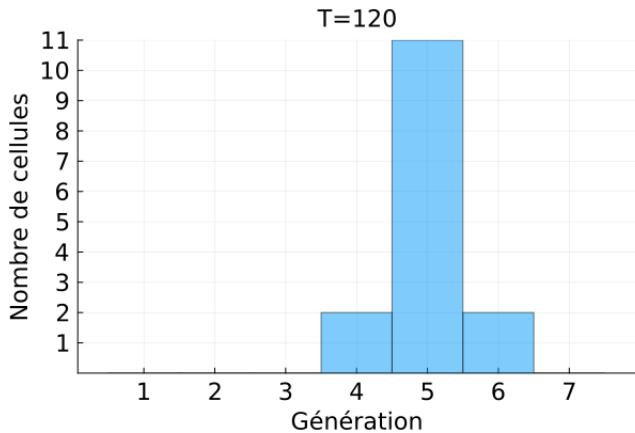


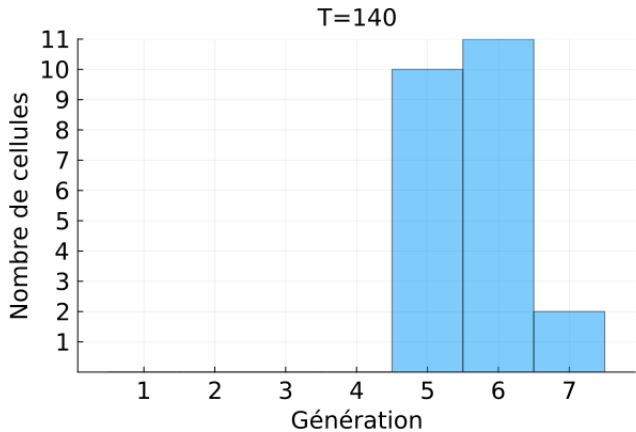


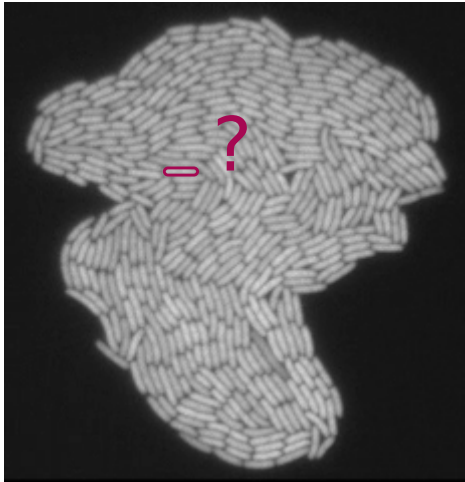












Durée du cycle cellulaire

- ◆ 20 min avec probabilité 0.5,
- ◆ 40 min avec probabilité 0.5.

Idée

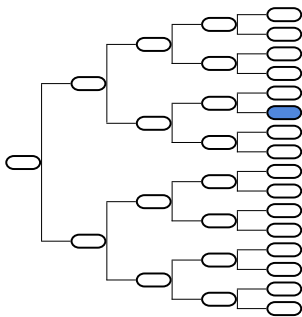
Modèle le long d'une lignée

Durée du cycle cellulaire

- ◆ 20 min avec probabilité 0.5,
- ◆ 40 min avec probabilité 0.5.

Idée

Modèle le long d'une lignée

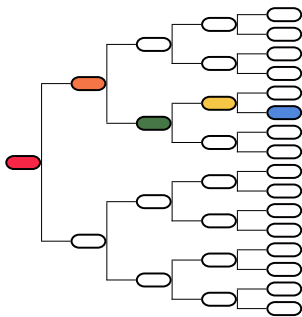


Durée du cycle cellulaire

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- ◆ 40 min avec probabilité 0.5.

Idée

Modèle le long d'une lignée

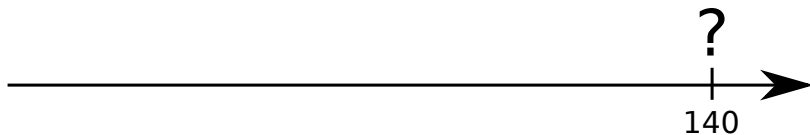


Exemple

Cycles cellulaires de 20 min ou 40 min.

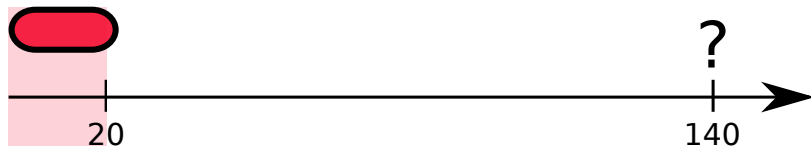
Exemple

Cycles cellulaires de 20 min ou 40 min.



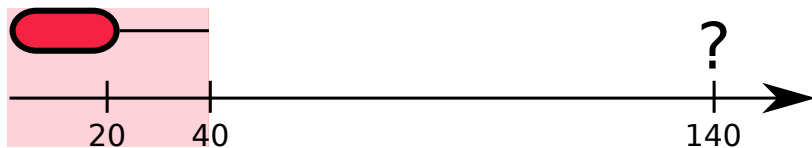
Exemple

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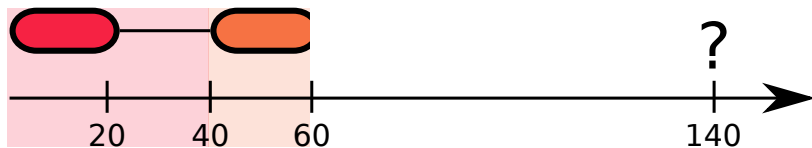
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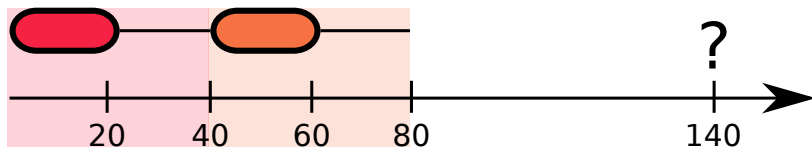
Exemple

Cycles cellulaires de 20 min ou 40 min.



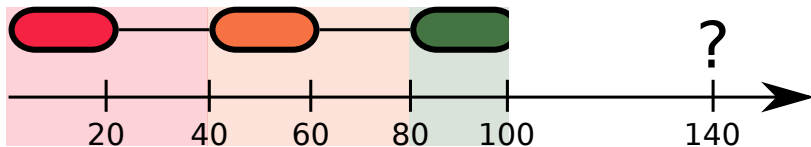
Exemple

Cycles cellulaires de 20 min ou 40 min.



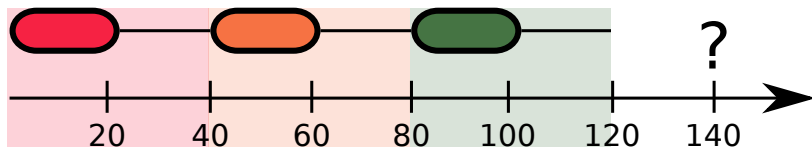
Exemple

Cycles cellulaires de 20 min ou 40 min.



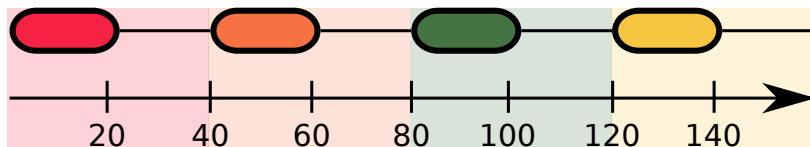
Exemple

Cycles cellulaires de 20 min ou 40 min.



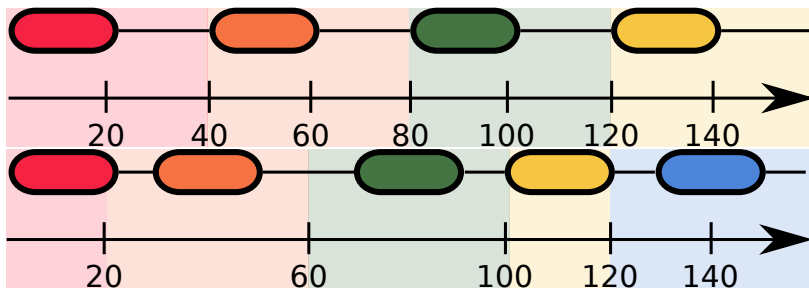
Exemple

Cycles cellulaires de 20 min ou 40 min.



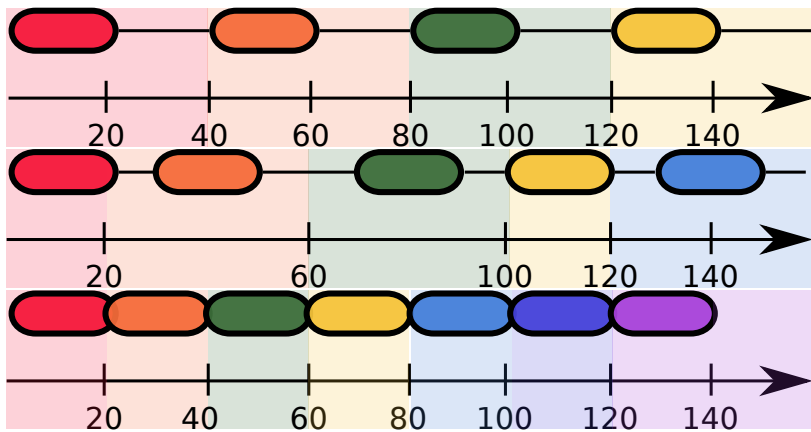
Exemple

Cycles cellulaires de 20 min ou 40 min.

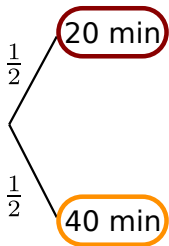


Exemple

Cycles cellulaires de 20 min ou 40 min.

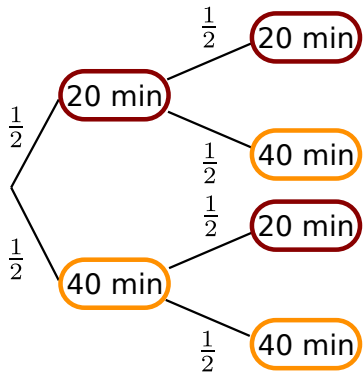


À $T = 30$,



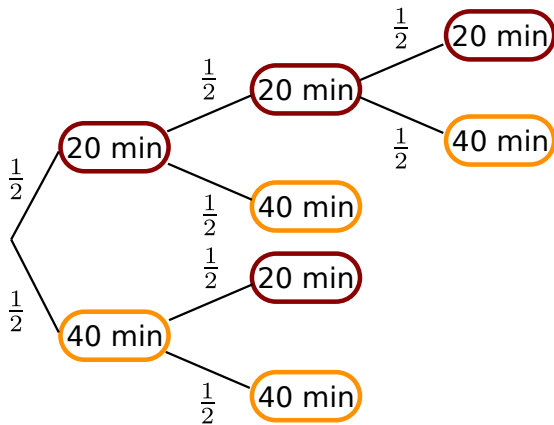
$$\mathbb{P}(G = 1) = 1/2, \mathbb{P}(G = 2) = 1/2$$

À $T = 50$,



$$\mathbb{P}(G = 1) = 0, \quad \mathbb{P}(G = 2) = 3/4, \quad \mathbb{P}(G = 3) = 1/4$$

À $T = 70$,



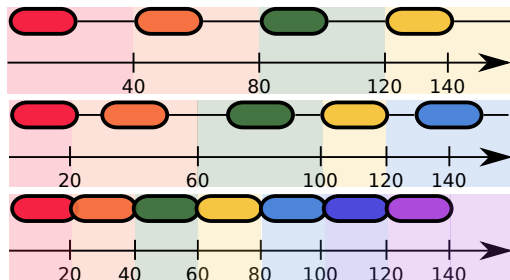
$$\mathbb{P}(G = 1) = 0, \quad \mathbb{P}(G = 2) = 1/4, \quad \mathbb{P}(G = 3) = 5/8, \quad \mathbb{P}(G = 4) = 1/8$$

Durée du cycle cellulaire

- ◆ 20 min avec probabilité 0.5,
- ◆ 40 min avec probabilité 0.5.

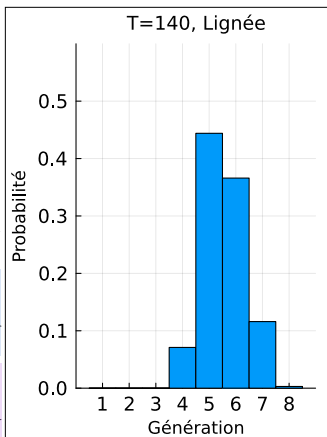
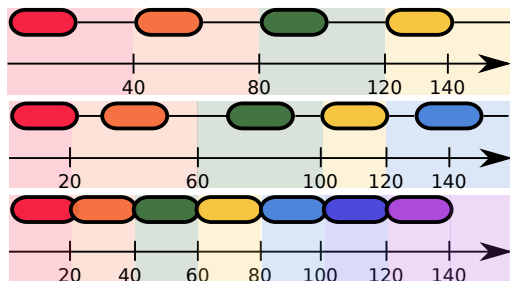
Durée du cycle cellulaire

- ◆ 20 min avec probabilité 0.5,
- ◆ 40 min avec probabilité 0.5.



Durée du cycle cellulaire

- ◆ 20 min avec probabilité 0.5,
- ◆ 40 min avec probabilité 0.5.



$$\mathbb{P}(G = 4) = 8/128, \quad \mathbb{P}(G = 5) = 56/128, \quad \mathbb{P}(G = 6) = 50/128,$$

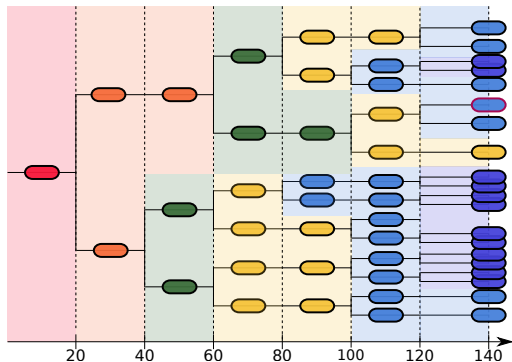
$$\mathbb{P}(G = 7) = 13/128, \quad \mathbb{P}(G = 8) = 1/128. \quad \mathbb{E}[G] = 5.6$$

Durée du cycle cellulaire

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- ◆ 40 min avec probabilité 0.5.

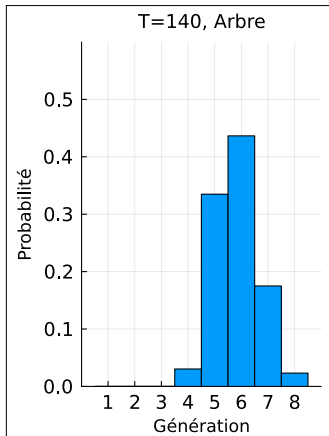
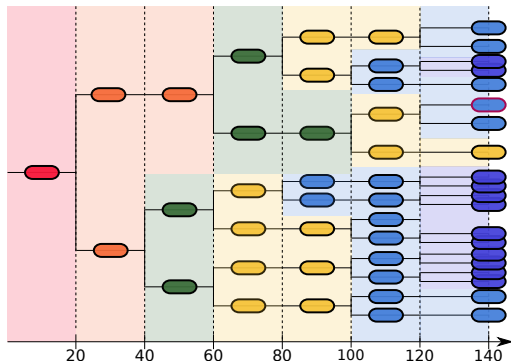
Durée du cycle cellulaire

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- ◆ 40 min avec probabilité 0.5.



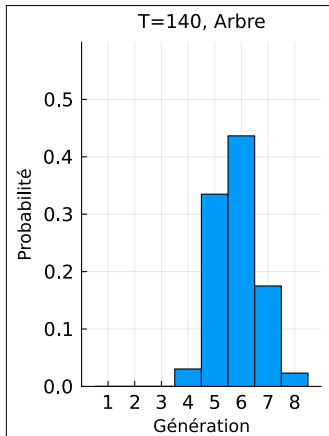
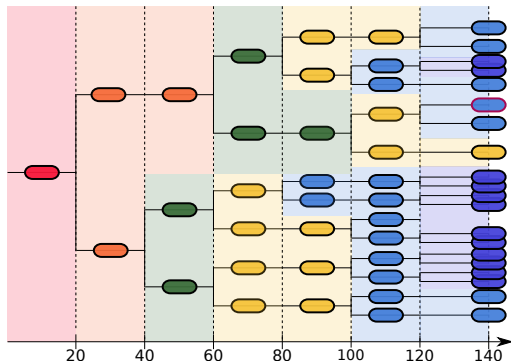
Durée du cycle cellulaire

- ◆ 20 min avec probabilité 0.5,
- ◆ 40 min avec probabilité 0.5.



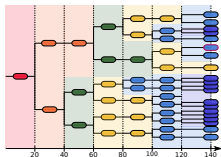
Durée du cycle cellulaire

- ◆ 20 min avec probabilité 0.5,
- ◆ 40 min avec probabilité 0.5.

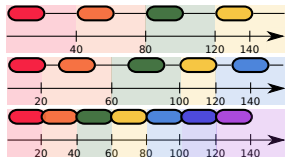
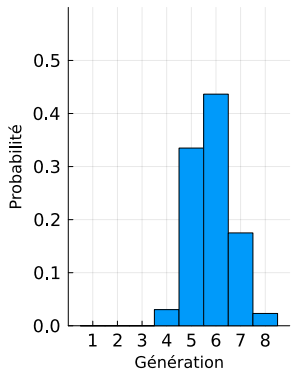


$$\mathbb{E}[G] = 5.9$$

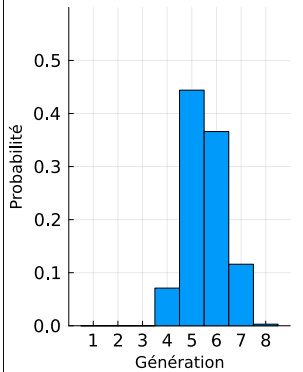
Comparaison



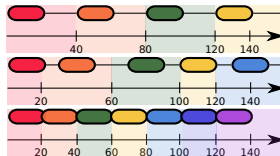
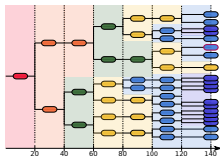
T=140, Arbre



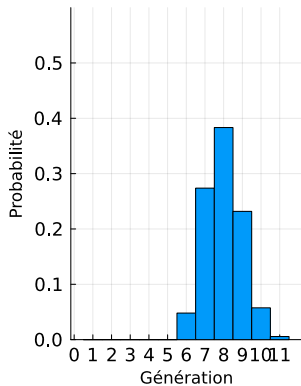
T=140, Lignée



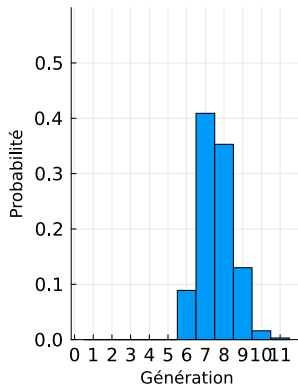
Comparaison en temps long

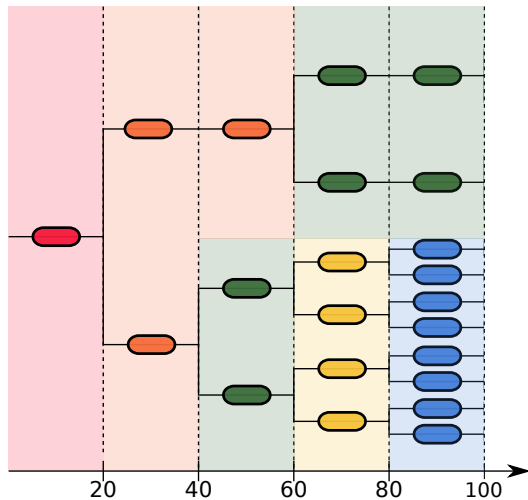


T=200, Arbre

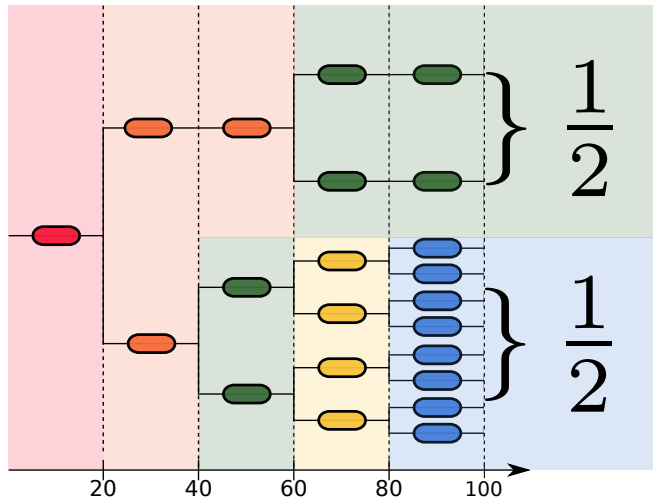


T=200, Lignée

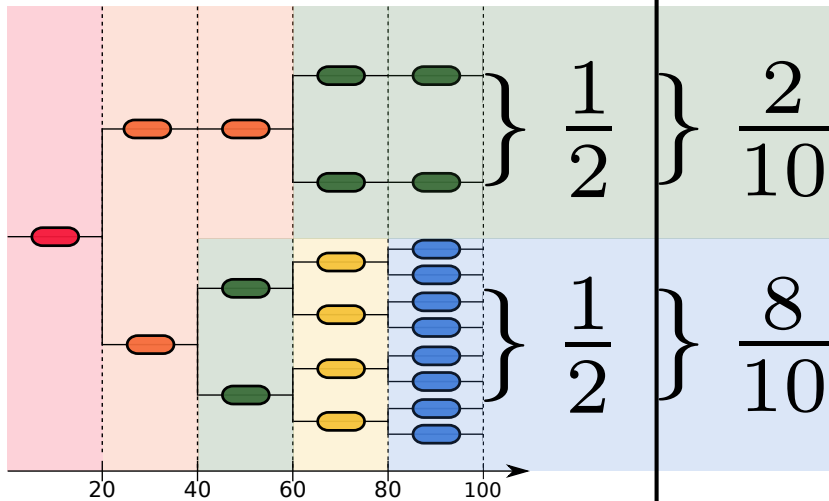




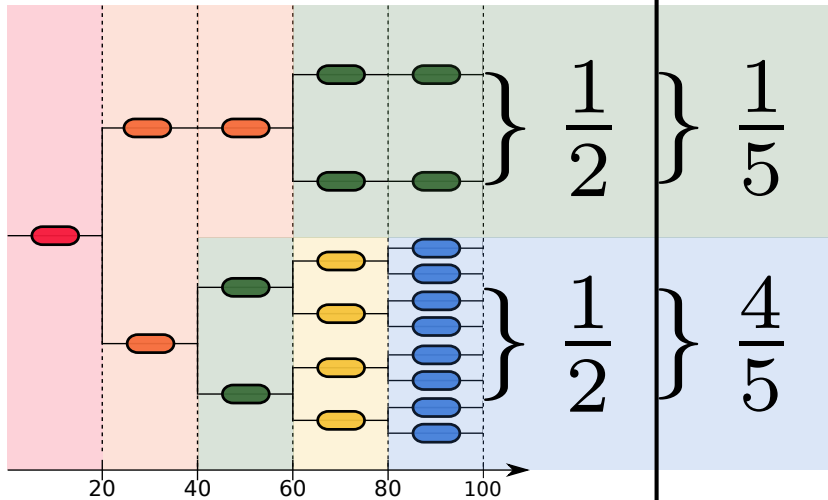
Lignée



Lignée | Arbre



Lignée | Arbre



Modèles mathématiques pour :

- ◆ Se substituer à des mesures fastidieuses,
- ◆ Comprendre des dynamiques complexes,
- ◆ Prévoir des comportements,
- ◆ etc.

Généralisations :

- ◆ N'importe quelle durée de vie,
- ◆ Mécanismes intracellulaires,
- ◆ Élongation,
- ◆ etc.

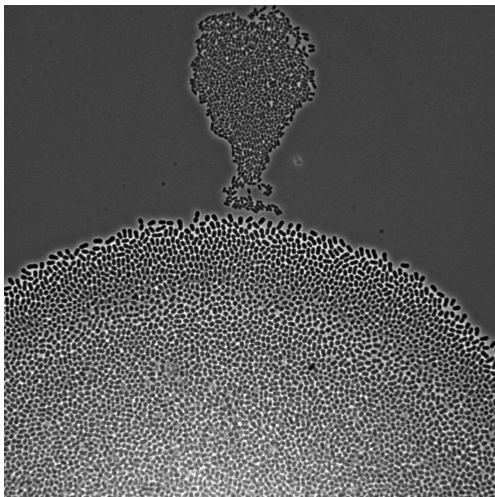


Figure: Montgolfière, par Antrea Pavlou

Merci pour votre attention !