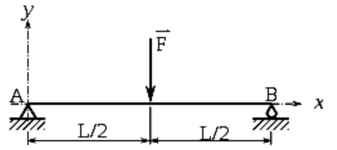
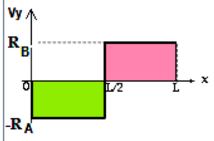
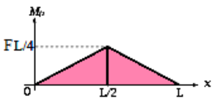
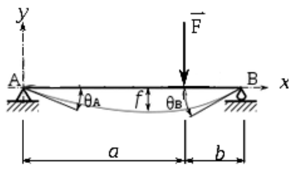
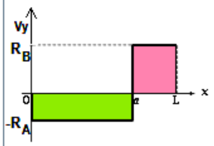
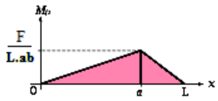
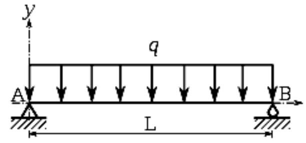
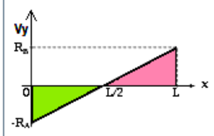
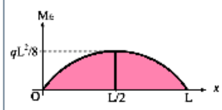
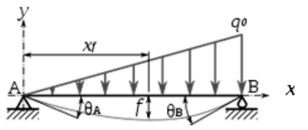
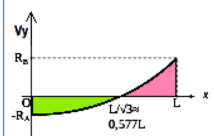
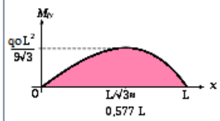
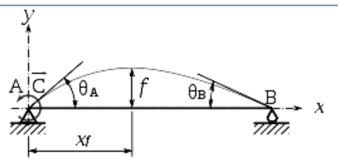
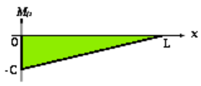
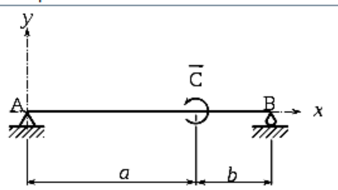
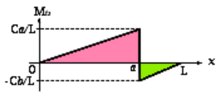
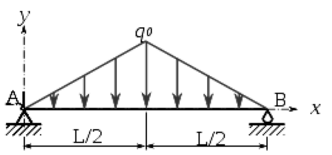
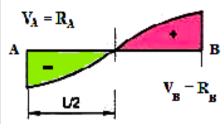
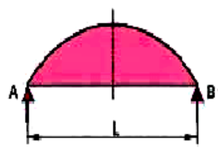
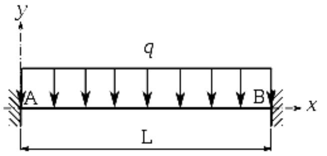
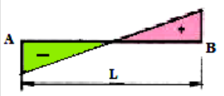
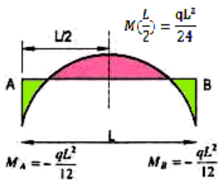
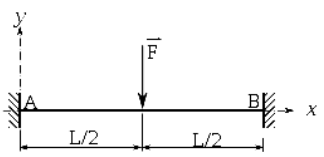
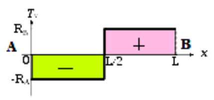
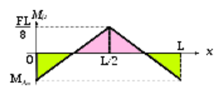
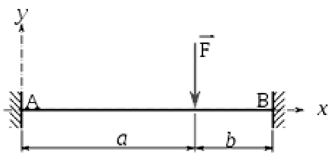
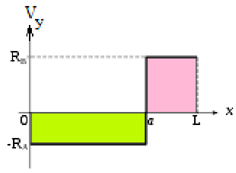
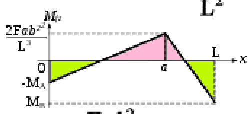
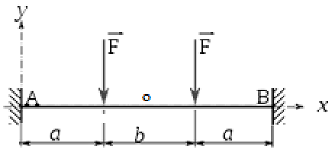
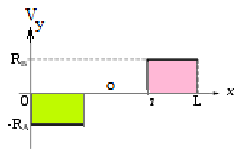
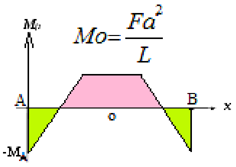
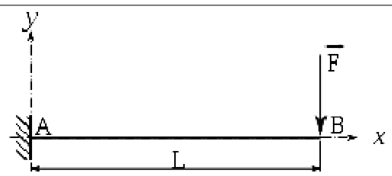
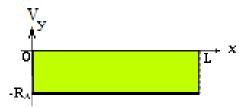
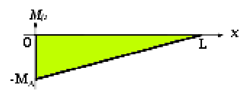
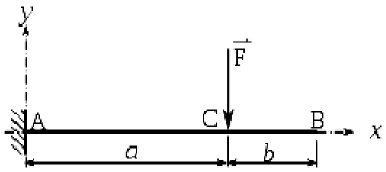
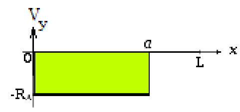
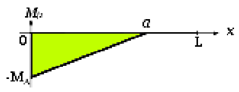
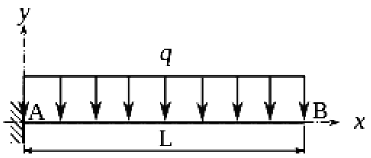
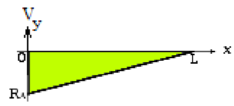
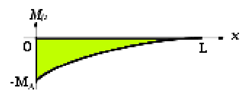
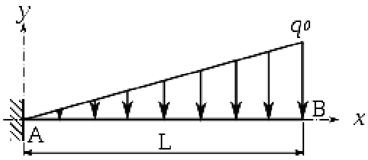
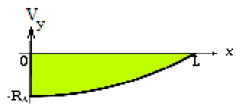
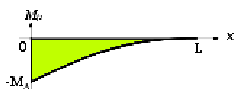
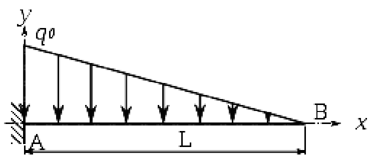
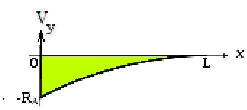
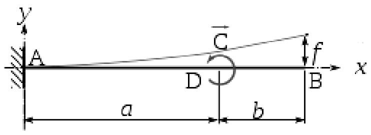
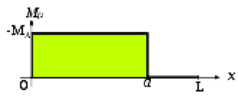
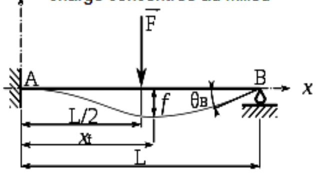
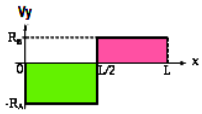
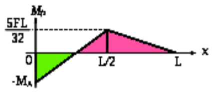
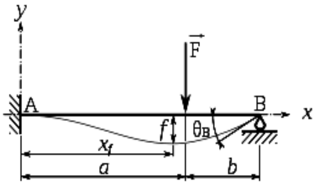
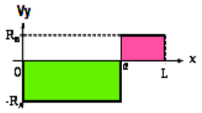
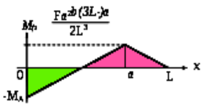
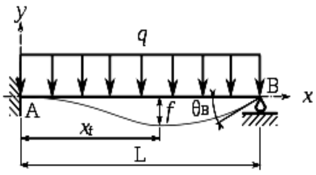
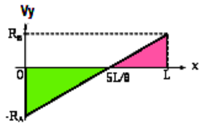
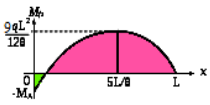
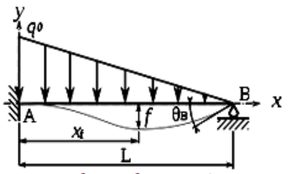
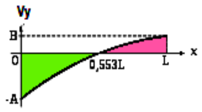
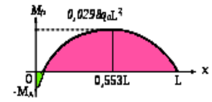
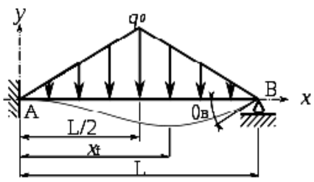
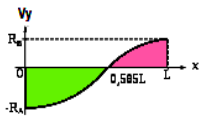
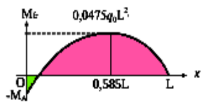
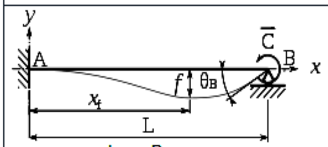
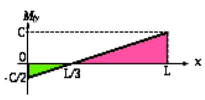
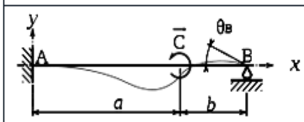
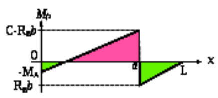
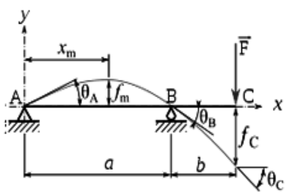
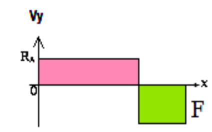
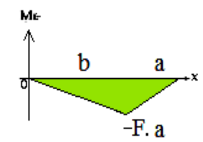
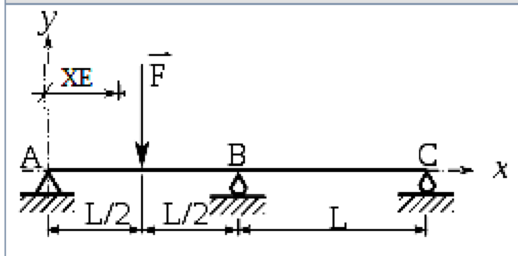
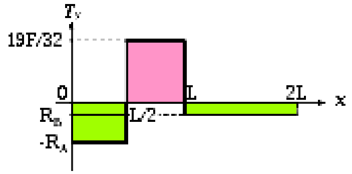
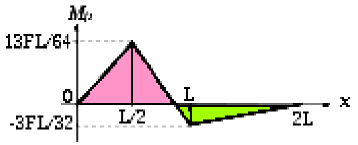
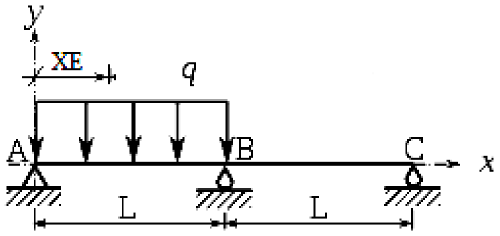
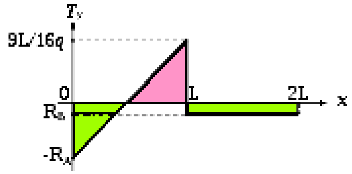
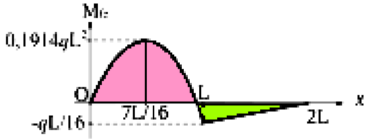
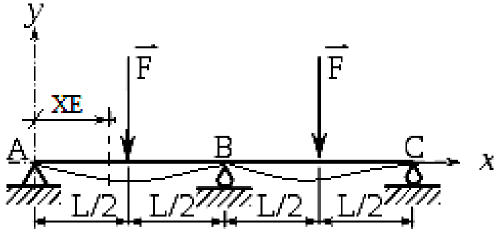
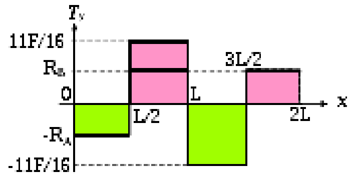
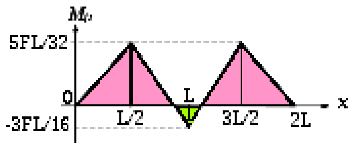
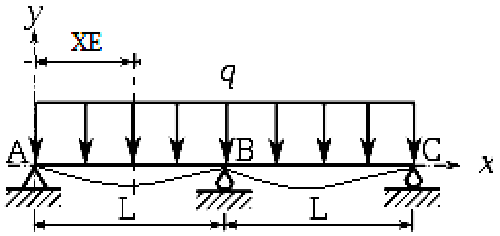
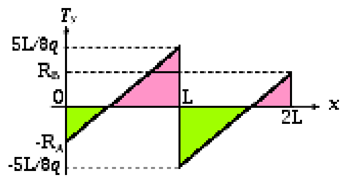
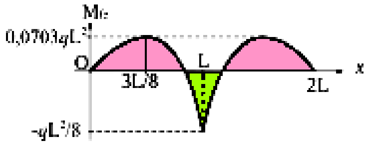
Génie Civil

Formulaires et Abaques RdM

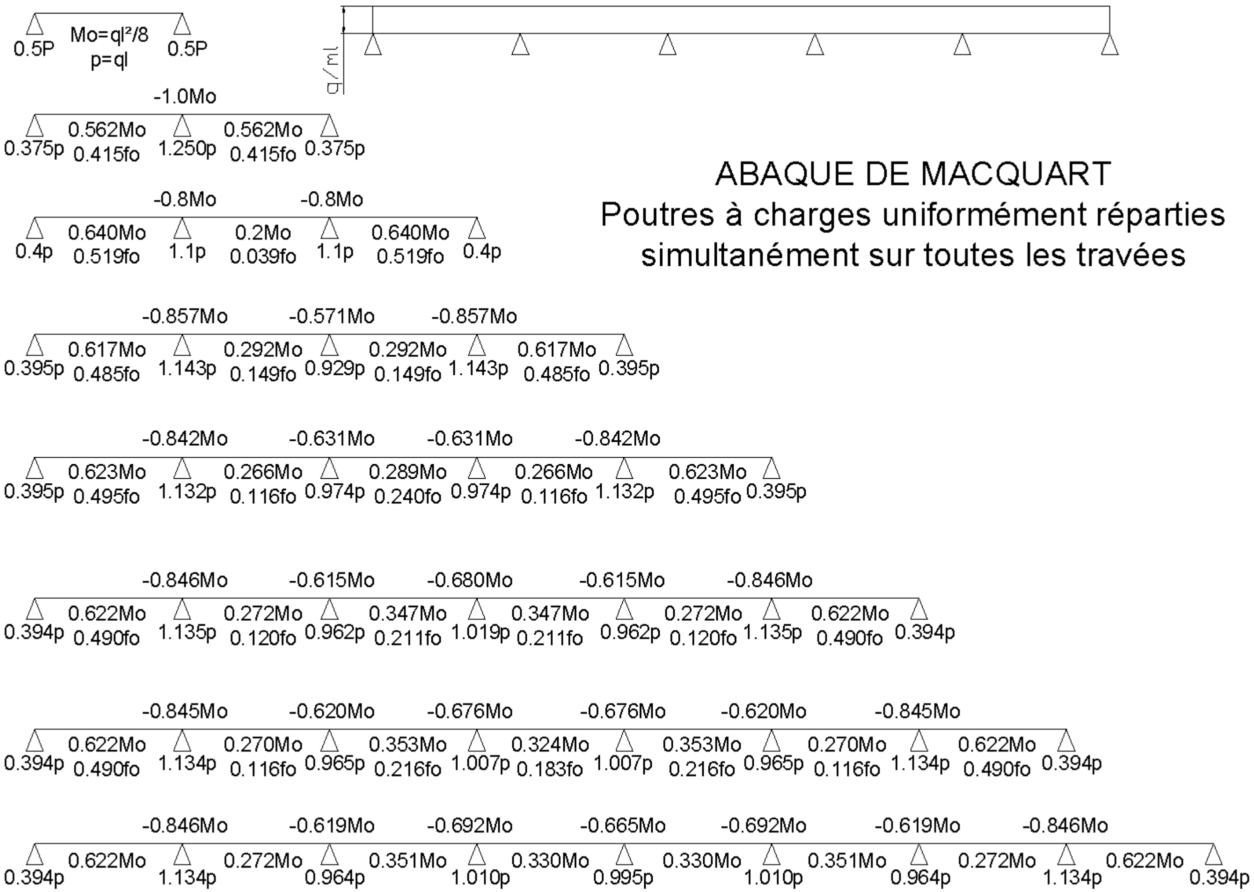
Sollicitation	Actions aux liaisons	Effort tranchant	Moment fléchissant	Flèche à l/2	Rotation aux appuis
 charge concentrée au centre (flexion trois points)	$R_A = R_B = \frac{F}{2}$			$\frac{FL^3}{48EI}$	$W_A = -\frac{FL^2}{16EI}$ $W_B = +\frac{FL^2}{16EI}$
 charge concentrée	$R_A = \frac{Fb}{L}$ $R_B = \frac{Fa}{L}$			$f_{l/2} = \frac{-Fb}{48EI}(3L^2-4b^2)$ $f_a = \frac{-Fa^2b^2}{3EIL}$ $f_{max} = \frac{-Fb(L^2-b^2)^{3/2}}{9L \times EI_{Gz}\sqrt{3}}$ position maxi $\sqrt{((L^2-b^2)/3)}$	$W_A = -\frac{Fb}{6EIL}(b^2-L^2)$ $W_B = \frac{Fa}{6EIL}(L^2-a^2)$
 charge uniforme	$R_A = R_B = \frac{qL}{2}$			$fl/2 = \frac{-5qL^4}{384EI}$	$W_A = \frac{-qL^3}{24EI}$ $W_B = \frac{+qL^3}{24EI}$
 charge linéaire croissante $q(x) = q_0x/L$	$R_A = \frac{p_1 L}{6}$ $R_B = \frac{p_1 L}{3}$			$f \simeq -\frac{0,00652 q_0 L^4}{384EI}$ $x_f \simeq 0,5193L$	$W_A = \frac{7q_0L^3}{360EI}$ $W_B = \frac{q_0L^3}{45EI}$
 couple concentré en A	$R_A = -R_B = \frac{C}{L}$			$f = \frac{CL^2}{16EI}$ $x_f = \left(1-\sqrt{\frac{1}{3}}\right)$ $L \simeq 0,423L$	$W_A = \frac{CL}{3EI}$ $W_B = -\frac{CL}{6EI}$
 couple concentré en $x = a$	$R_A = -R_B = \frac{C}{L}$			$f_a = -\frac{C.ab}{3EIL}(a-b)$ $f_{L/2} = -\frac{C}{16EI}(4a^2-L^2)$	$w_A = \frac{-C(6aL-3a^2-2L^2)}{6.E.I.L}$ $w_B = \frac{+C(3a^2-L^2)}{6.E.I.L}$
 charge triangulaire symétrique	$R_A = R_B = \frac{qL}{4}$	 $V(x) = \frac{qL}{4}\left[1-4\frac{x^2}{L^2}\right]$ pour $x \le \frac{L}{2}$	 $M_0 = \frac{qL^2}{12}$ pour $x_0 = \frac{L}{2}$	$f_{max} = -\frac{q.L^4}{120EI}$	$W_A = -\frac{5\ q.L^3}{192EI}$ $W_B = +\frac{5\ q.L^3}{192EI}$
 charge uniforme	$R_A = R_B = \frac{qL}{2}$	 $V_A = R_A$ $V_B = R_B$		$Mo = \frac{qL^2}{24}$ $f\,max = \frac{-qL^4}{384.EI}$	$w_A = 0$ $w_B = 0$
 charge concentrée au centre	$R_A = R_B = \frac{F}{2}$ $M_A = -M_B = \frac{FL}{8}$	 $V_A = R_A$ $V_B = R_B$		$Mo = \frac{FL}{8}$ $f\,max = \frac{FL^3}{192EI}$	$w_A = 0$ $w_B = 0$

Sollicitation	Actions aux liaisons	Effort tranchant	Moment fléchissant	Flèche
 charge excentrée (p. ex. charge roulante)	$R_A = \frac{Fb^2(3a+b)}{L^3}$ $R_B = \frac{Fa^2(a+3b)}{L^3}$		$M_B = -\frac{Fa^2b}{L^2}$  $M_A = \frac{Fab^2}{L^2}$	$f(a) = -\frac{Fa^3b^3}{3EIL^3}$
 2 charges ponctuelles	$R_A = F$ $R_B = F$ $MA = \frac{Fa(L-a)}{L}$		$Mo = \frac{Fa^2}{L}$  $MA = \frac{Fa(L-a)}{L}$	$f(a) = -\frac{Fa^3(2L-3a)}{6EIL}$ $f(o) = -\frac{Fa^2(3L-4a)}{24EI}$
 charge concentrée à l'extrémité	$R_A = F$ $M_A = FL$			$f\max = \frac{-FL^3}{3EI}$
 charge concentrée	$R_A = F$ $M_A = Fa$			$f\max = \frac{-F(L-a)^2 \times (2L+a)}{6EI}$
 charge uniforme	$R_A = qL$ $M_A = \frac{qL^2}{2}$			$f\max = \frac{-qL^4}{8EI}$
 charge croissante $q(x) = q_0 x/L$	$R_A = \frac{q_0 L}{2}$ $M_A = \frac{q_0 L^2}{3}$			$f = -\frac{11 q_0 L^4}{120EI}$
 charge décroissante $q(x) = q_0(1-x/L)$	$R_A = \frac{q_0 L}{2}$ $M_A = \frac{q_0 L^2}{6}$			$f\max = -\frac{q0 \times L^4}{30EI}$
 couple	$R_A = 0$ $M_A = -C$			$f = \frac{Ca}{EI}(L - \frac{a}{2})$

Sollicitation	Actions aux liaisons	Effort tranchant	Moment fléchissant	Flèche	Rotation aux appuis
charge concentrée au milieu 	$R_A = \frac{11F}{16}$ $M_A = \frac{3FL}{16}$ $R_B = \frac{5F}{16}$			$y\left(\frac{L}{2}\right) = -\frac{7FL^3}{768EI}$ $x_f \simeq 0,5525L$	$\theta_B = \frac{FL^2}{32EI}$
 charge concentrée en $x = a$	$R_A = \frac{Fb(3L^2-b^2)}{2L^3}$ $M_A = \frac{Fa(L^2-a^2)}{2L^2}$ $R_B = \frac{Fa^2(2L+b)}{2L^3}$			$y(a) = -\frac{Fb(3L^2-5b^2)b^2}{96EI}$ $a < 0,5859L$: $x_f > a$ $a > 0,5859L$: $x_f < a$	$\theta_B = \frac{Fa^2b}{4EIL}$
 charge uniforme	$R_A = \frac{5qL}{8}$ $M_A = \frac{qL^2}{8}$ $R_B = \frac{3qL}{8}$			$f \simeq \frac{qL^4}{185\ EI}$ $x_f \simeq 0,578L$	$\theta_B = \frac{5qL^3}{348EI}$
 charge linéaire décroissante $q(x) = q_0(1 - x/L)$	$R_A = \frac{2q_0L}{5}$ $M_A = \frac{q_0L^2}{15}$ $R_B = \frac{q_0L}{10}$			$f \simeq -0,00239\frac{q_0L^4}{EI}$ $x_f \simeq 0,553L$	$\theta_B = \frac{q_0L^3}{120EI}$
 charge triangulaire symétrique	$R_A = \frac{21q_0L}{64}$ $M_A = \frac{5q_0L^2}{64}$ $R_B = \frac{11q_0L}{64}$			$f \simeq \frac{0,00357q_0L^4}{EI}$ $x_f \simeq 0,57L$	$\theta_B = \frac{5q_0L^3}{384EI}$
 couple en B	$R_A = -R_B = \frac{3C}{2L}$ $M_A = \frac{C}{2}$			$f = -\frac{CL^2}{27EI}$ $x_f = \frac{2}{3}L$	$\theta_B = \frac{CL}{4EI}$
 couple en $x = a$	$R_A = -R_B = \frac{3C(L^2-b^2)}{2L^3}$ $M_A = \frac{C(L^2-3b^2)}{2L^2}$				
 force concentrée à l'extérieur des appuis	$R_A = \frac{-F.a}{b}$ $R_B = \frac{F.L}{b}$ $M_A = -F.a$			$f_m = -\frac{Fba^2}{9\sqrt{3}EI}$ $x_m = 0,577 \cdot a$ $f_C = \frac{Fb^2}{3EI}(a+b)$ $x \leqslant a$: $y = \frac{Fba^2}{6EI}\left(\frac{x^3}{a^3} - \frac{x}{a}\right)$	$\theta_A = \frac{Fab}{6EI}$ $\theta_B = \frac{Fab}{3EI}$ $\theta_C = \frac{Fb(2a+3b)}{6EI}$

Sollicitation	Actions aux liaisons	Effort tranchant	Moment fléchissant	flèche
 charge concentrée au milieu d'une travée	$R_A = \frac{13F}{32}$ $R_B = \frac{11F}{16}$ $R_C = \frac{-3F}{32}$			Flèche pour XE = 0,48 L $f_E = \frac{-3\ qL^3}{200\ E.I}$
 charge uniforme sur une travée	$R_A = \frac{7qL}{16}$ $R_B = \frac{15qL}{24}$ $R_C = \frac{-3qL}{48}$			Flèche pour XE = 0,472 L $f_E = \frac{-3.51\ qL^4}{384\ E.I}$
 charge concentrée au milieu de chaque travée	$R_A = R_C = \frac{5F}{16}$ $R_B = \frac{11F}{8}$			$X_E = \frac{L\sqrt{5}}{5}$ $f_E = -\frac{FL^3}{107\ EI}$
 charge uniforme sur toute la poutre	$R_A = R_C = \frac{3qL}{8}$ $R_B = \frac{5qL}{4}$			Flèche pour XE = 0,42 L $f_E = \frac{-2{,}05\ qL^4}{384\ E.I}$

ABAQUE DE MACQUART

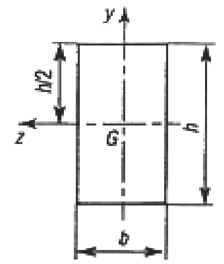


ABAQUE DE MACQUART
Poutres à charges uniformément réparties simultanément sur toutes les travées

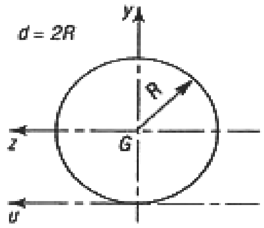
dans cette abaque on calcule le moment maximum Mo, les réactions et la flèche maximum de la travée simple considérée comme isostatique, puis on applique les coefficients donnés ci-dessus pour trouver les différents moments, flèches et réactions des poutres hyperstatiques

<u>nota :</u> le chargement est considéré comme une CUR uniformément répartie sur toute la longueur.

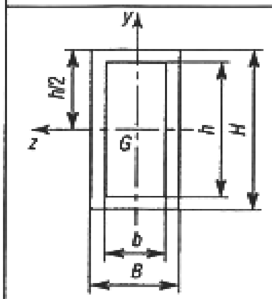
FORMULAIRE Moments quadratiques



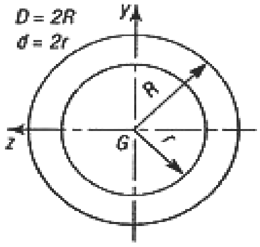
$$S = b\,h$$
$$I_z = \frac{b\,h^3}{12}$$
$$I_y = \frac{h\,b^3}{12}$$
$$I_G = \frac{b\,h}{12}\left(h^2 + b^2\right) = I_y + I_z$$



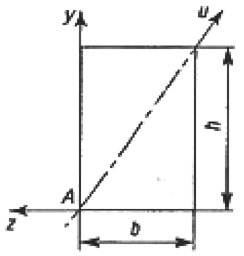
$$S = \pi R^2$$
$$I_y = I_z = \frac{\pi R^4}{4} = \frac{\pi d^4}{64}$$
$$I_u = \frac{5\,\pi R^4}{4} = \frac{5\,\pi d^4}{64}$$
$$I_G = \frac{\pi R^4}{2} = \frac{\pi d^4}{32} = I_y + I_z\,.$$



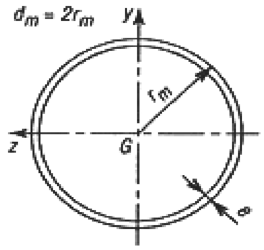
$$S = BH - bh$$
$$I_y = \frac{HB^3 - hb^3}{12}$$
$$I_z = \frac{BH^3 - bh^3}{12} = I_y + I_z$$



$$S = \pi\left(R^2 - r^2\right)$$
$$I_y = I_z = \frac{\pi}{4}\left(R^4 - r^4\right)$$
$$= \frac{\pi}{64}\left(D^4 - d^4\right)$$
$$I_G = \frac{\pi}{2}\left(R^4 - r^4\right)$$
$$= \frac{\pi}{32}\left(D^4 - d^4\right) = I_y + I_z$$



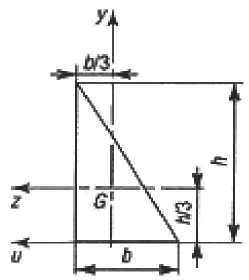
$$I_y = \frac{hb^3}{3}$$
$$I_z = \frac{bh^3}{3}$$
$$I_u = \frac{b^3\,h^3}{6\left(b^2 + h^2\right)}$$
$$I_A = \frac{b\,h}{3}\left(h^2 + b^2\right) = I_y + I_z$$



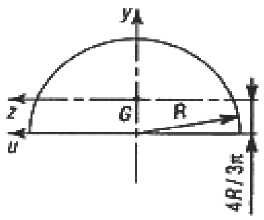
cas où *e* est faible

formules approchées

$$S = 2\,\pi\,r_m\,e = \pi\,e\,d_m$$
$$I_y = I_z = \pi\,e\,r_m^3 = \frac{\pi}{8}\,e\,d_m^3$$
$$I_G = 2\,\pi\,e\,r_m^3 = \frac{\pi}{4}\,e\,d_m^3$$

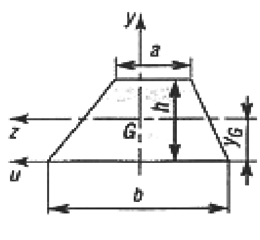


$$S = \frac{bh}{2}$$
$$I_y = \frac{h\,b^3}{36}$$
$$I_z = \frac{b\,h^3}{36}$$
$$I_u = \frac{b\,h^3}{12}$$
$$I_G = \frac{b\,h}{36}\left(h^2 + b^2\right) = I_y + I_z$$

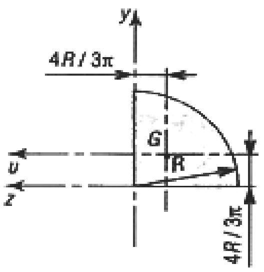


1/2 cercle

$$S = \frac{\pi R^2}{2}$$
$$I_y = \frac{\pi R^4}{8}$$
$$I_z \simeq 0{,}1098\ R^4$$
$$I_u = \frac{\pi R^4}{8}$$

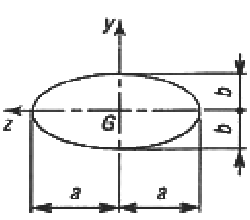


$$S = \frac{h}{2}\left(a + b\right)$$
$$y_G = \frac{h\left(2a + b\right)}{3\left(a + b\right)}$$
$$I_z = \frac{h^3\left(a^2 + 4ab + b^2\right)}{36\left(a + b\right)}$$
$$I_u = \frac{h^3\left(3a + b\right)}{12}$$



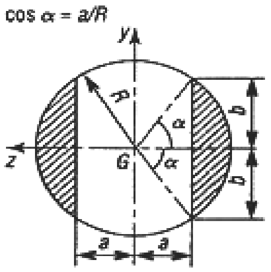
1/4 de cercle

$$S = \frac{\pi R^2}{4}$$
$$I_y = I_z = \frac{\pi R^4}{16}$$
$$I_u = 0{,}05488\ R^4$$



ellipse

$$S = \pi\,a\,b$$
$$I_z = \frac{\pi\,a\,b^3}{4}$$
$$I_y = \frac{\pi\,b\,a^3}{4}$$
$$I_G = \frac{\pi\,a\,b}{4}\left(a^2 + b^2\right) = I_z + I_y$$



cercle + trou de perçage

$b = \sqrt{R^2 - a^2}$; α en radian

$$S = 2R^2\left(\alpha - \frac{a\,b}{R^2}\right)$$
$$I_z = \frac{R^4}{6}\left(3\alpha - \frac{3\,ab}{R^2} - \frac{2\,ab^3}{R^4}\right)$$
$$I_y = \frac{R^4}{2}\left(\alpha - \frac{ab}{R^2} + \frac{2\,ab^3}{R^4}\right)$$

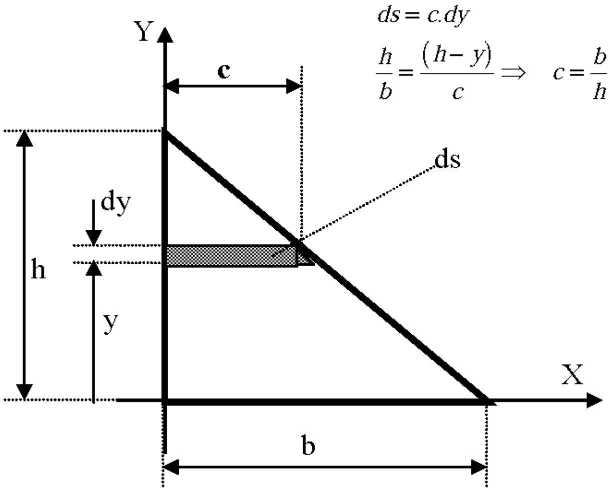
Moments d 'inerties et produit d'inertie des sections

1: Triangle:

$$I_x = \int_s y^2 \cdot ds = \frac{b}{h}\int_0^h y^2\,(h-y)\,dy$$

$$= \frac{b}{h}\int_0^h \left(y^2 h - y^3\right) dy = \frac{b}{h}\left[h\frac{y^3}{3} - \frac{y^4}{4}\right]_0^h = \frac{bh^3}{12}$$

$$I_{Gx} = \frac{bh^3}{12} - \left(\frac{h}{3}\right)^2 \cdot \frac{bh}{2} = \frac{bh^3}{36}$$



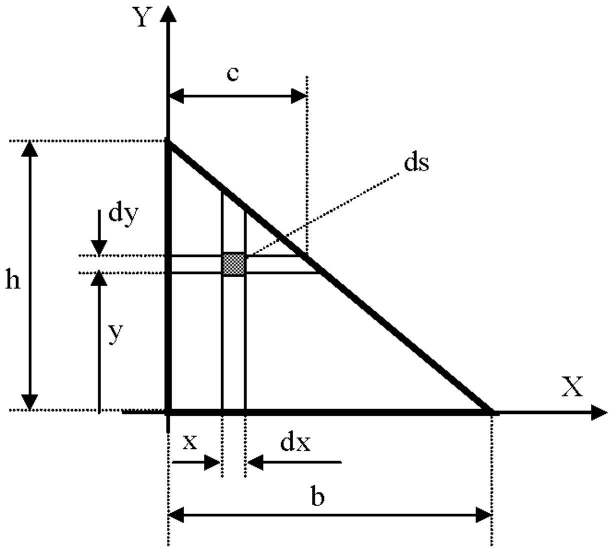
<u>Produit d'inertie</u>:

$$I_{xy} = \int_s xyds = \int_0^h ydy\int_0^c xdx$$

$$I_{xy} = \int_0^h ydy \cdot \frac{c^2}{2} = \frac{b^2}{2h^2}\int_0^h y(h-y)^2\,dy$$

$$I_{xy} = \frac{b^2}{2h^2}\left[\int_0^h \left(yh^2 + y^3 - 2hy^2\right) dy\right] = \frac{b^2h^2}{24}$$

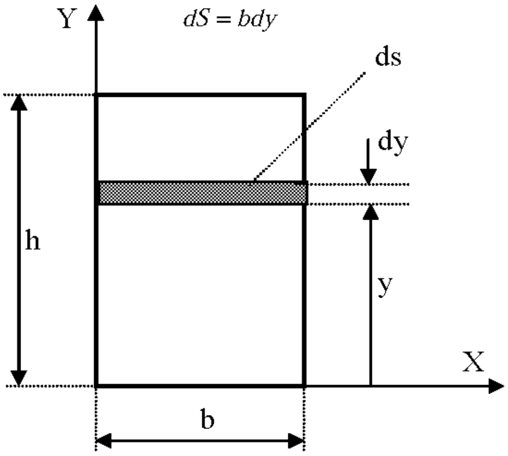
$$I_{GxGy} = \frac{b^2h^2}{24} - \left(\frac{h}{3}\right)\left(\frac{b}{3}\right)\left(\frac{bh}{2}\right) = \frac{b^2h^2}{24} - \frac{b^2h^2}{18} = -\frac{b^2h^2}{72}$$



2: Rectangle:

$$I_x = \int_s y^2\,ds = b\int_0^h y^2\,dy = b\left[\frac{y^3}{3}\right]_0^h = \frac{bh^3}{3}$$

$$I_{Gx} = \frac{bh^3}{3} - \left(\frac{h}{2}\right)^2 bh = \frac{bh^3}{12}$$



3: SURFACE CIRCULAIRE

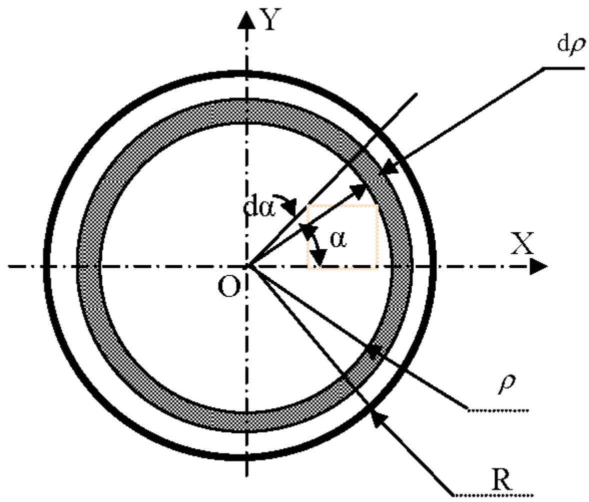
$$I_x = \int_s y^2 ds = \int_0^R \int_0^\alpha \rho^3 \sin^2 \alpha \, d\rho \, d\alpha$$

$$I_x = \int_0^R \rho^3 d\rho \int_0^\alpha \sin^2\alpha \, d\alpha = \frac{R^4}{4} \int_0^\alpha \sin^2 \alpha \, d\alpha$$

$ds = \rho d\alpha d\rho$

$y = \rho \sin\alpha$

$x = \rho \cos\alpha$



a) Si **α = 2π** => $I_x = I_y = \dfrac{\pi R^4}{4} = \dfrac{\pi D^4}{64}$

$$I_x = I_y = I_{Gx} = I_{Gy} = \frac{\pi R^4}{4} = \frac{\pi D^4}{64}$$

b) Si **α = π** => $I_x = I_y = \dfrac{\pi R^4}{8} = \dfrac{\pi D^4}{128}$

- $I_{Gx} = I_x - \left(\dfrac{4R}{3\pi}\right)^2 \dfrac{\pi R^2}{2} = \dfrac{\pi R^4}{8} - \dfrac{8\pi R^4}{9}$
- $I_{Gy} = I_y = \dfrac{\pi R^4}{8} = \dfrac{\pi D^4}{128}$

c) Si **α = π /2** => $I_x = I_y = \dfrac{\pi R^4}{16} = \dfrac{\pi D^4}{256}$

$$I_{Gx} = I_{Gy} = \frac{\pi R^4}{16} - \left(\frac{4R}{3\pi}\right)^2 \frac{\pi R^2}{4} = \frac{\pi R^4}{16} - \frac{4\pi R^4}{9}$$

Produit d'inertie:

Formule trigonométrique:

$$2\sin\alpha\cos\alpha = \sin 2\alpha$$

$$\int \sin 2\alpha \, d\alpha = -\frac{1}{2}\cos 2\alpha$$

$$Ixy = \int_s \rho^3 \sin\alpha \cos\alpha \, d\alpha \, d\rho = \frac{1}{2}\left[\frac{\rho^4}{4}\right]_0^R \int_0^\alpha \sin 2\alpha \, d\alpha$$

$$Ixy = -\frac{R^4}{16}\left[\cos 2\alpha\right]_0^\alpha$$

$$si \begin{cases} \alpha=0 \ \Rightarrow Ixy = -\dfrac{R^4}{16}[1-1] = 0 \\ \alpha=\dfrac{\pi}{2} \ \Rightarrow Ixy = -\dfrac{R^4}{16}[0-1] = \dfrac{R^4}{16} \\ \alpha=\pi \ \Rightarrow Ixy = -\dfrac{R^4}{16}[1-1] = 0 \end{cases}$$